GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P4

Paper G

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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P4 Paper G – Marking Guide

1.		$\frac{x^2-12}{x} - 1 \geq 0 \;\therefore\; \frac{x^2-x-12}{x} \geq 0$	M1 A1	
		$\frac{(x-4)(x+3)}{x} \geq 0 \;\therefore$ critical values are $-3, 0, 4$	M1 A1	
		considering change of sign of factors and expression undefined at $x = 0$ gives		
		$-3 \leq x < 0$ or $x \geq 4$	M1 A2	**(7)**
2.		$5^2 + 9^2 + 13^3 + 17^2 + \ldots = \sum_{r=1}^{n} (4r+1)^2 = \sum_{r=1}^{n} (16r^2 + 8r + 1)$	M1 A2	
		$= 16 \times \frac{1}{6}n(n+1)(2n+1) + 8 \times \frac{1}{2}n(n+1) + n$	M1 A1	
		$= \frac{1}{3}n[8(2n^2 + 3n + 1) + 12(n+1) + 3]$	M1	
		$= \frac{1}{3}n(16n^2 + 36n + 23)$	A1	**(7)**
3.	*(a)*	$f(0) = 2;\; f(1) = -2$		
		f cont. over interval, change of sign $\therefore$ root	M1 A1	
	(b)	$f'(x) = 3x^2 - 10x$	M1	
		$x_{n+1} = x_n - \frac{x_n^3 - 5x_n^2 + 2}{3x_n^2 - 10x_n}$	M1 A1	
		giving $\alpha = 0.68$ (2dp)	M1 A1	
	(c)	e.g. $f'(0) = 0 \;\therefore$ tangent to curve at $x = 0$ is parallel to x-axis		
		and N-R uses intersection of tangent with x-axis as next approx.	B2	**(9)**
4.	*(a)*	$z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}} \times \frac{1+i\sqrt{3}}{1+i\sqrt{3}} = \frac{-2+2i\sqrt{3}}{4}$	M1 A1	
		$= \frac{-1+i\sqrt{3}}{2} = -\frac{1}{2}(1 - i\sqrt{3}) \;\therefore\; \lambda = -\frac{1}{2}$	M1 A1	
	(b)	$\text{mod } z = \frac{1}{2}\sqrt{(1+3)} = 1$	B1	
		$\arg z = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = \frac{2\pi}{3}$	M1 A1	
	(c)	$\text{mod } z^4 = 1^4 = 1$	M1 A1	
		$\arg z^4 = 4 \arg z = \frac{8\pi}{3}$ (or $\frac{2\pi}{3}$)	M1 A1	**(11)**
5.	*(a)*	$\frac{dy}{dx} = p\cos x - q\sin x,\; \frac{d^2y}{dx^2} = -p\sin x - q\cos x$	M1 A1	
		$-p\sin x - q\cos x + 2p\cos x - 2q\sin x + 5p\sin x + 5q\cos x = \sin x$	M1 A1	
		$4p - 2q = 1$		
		$2p + 4q = 0$	A1	
		giving $p = \frac{1}{5},\; q = -\frac{1}{10}$	M1 A1	
	(b)	aux. eqn. $m^2 + 2m + 5 = 0$	M1	
		$m = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$	M1 A1	
		$\therefore\; y = e^{-x}(A\cos 2x + B\sin 2x) + \frac{1}{5}\sin x - \frac{1}{10}\cos x$	M1 A1	**(12)**

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6.	*(a)*	$\int 2\cot x \; dx = \int \frac{2\cos x}{\sin x} \; dx$	M1	
		$= 2\ln\lvert \sin x \rvert + c$	A1	
		$= \ln(\sin^2 x) + c \qquad [\sin^2 x \geq 0]$	A1	
	(b)	$\frac{dy}{dx} + 2y\cot x = \operatorname{cosec} x$	M1	
		int. fac. $= e^{\int 2\cot x \, dx} = e^{\ln(\sin^2 x)} = \sin^2 x$	M1	
		$\therefore \sin^2 x \frac{dy}{dx} + 2y\sin x\cos x = \sin x$	A1	
		$\frac{d}{dx}(y\sin^2 x) = \sin x$	M1	
		$y\sin^2 x = \int \sin x \; dx$		
		$y\sin^2 x = -\cos x + c$	A1	
	(c)	$x = \frac{\pi}{4}, y = 0 \;\; \therefore c = \frac{1}{\sqrt{2}}$ so $y\sin^2 x = \frac{1}{\sqrt{2}} - \cos x$	M1 A1	
		$\therefore$ when $x = \frac{\pi}{3}$, $(\frac{\sqrt{3}}{2})^2 y = \frac{1}{\sqrt{2}} - \frac{1}{2}$	M1	
		$\therefore \frac{3}{4}y = \frac{1}{2}(\sqrt{2} - 1)$ giving $y = \frac{2}{3}(\sqrt{2} - 1)$	A1	**(12)**
7.	*(a)*	$2(1 + \cos\theta)\cos\theta = \frac{3}{2}$	M1	
		$4\cos^2\theta + 4\cos\theta - 3 = 0$	A1	
		$(2\cos\theta - 1)(2\cos\theta + 3) = 0$	M1	
		$\cos\theta = -\frac{3}{2}$ (no solns) or $\frac{1}{2}$	A1	
		$\therefore \theta = \pm\frac{\pi}{3}$	A1	
		giving $A(3, \frac{\pi}{3})$ and $B(3, -\frac{\pi}{3})$	A1	
	(b)	$\angle AOB = \frac{2\pi}{3}$	B1	
		area of triangle $OAB = \frac{1}{2} \times 3 \times 3 \times \sin\frac{2\pi}{3}$	M1	
		$= \frac{1}{2} \times 9 \times \frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{4}$	A1	
	(c)	area between OA, curve and x-axis $= \frac{1}{2}\int_0^{\frac{\pi}{3}} 4(1 + \cos\theta)^2 \; d\theta$	M1	
		$= \int_0^{\frac{\pi}{3}} 2 + 4\cos\theta + 2\cos^2\theta \; d\theta$	A1	
		$= \int_0^{\frac{\pi}{3}} 3 + 4\cos\theta + \cos 2\theta \; d\theta$	M1	
		$= [\,3\theta + 4\sin\theta + \frac{1}{2}\sin 2\theta\,]_0^{\frac{\pi}{3}}$	A1	
		$= (\pi + 4\times\frac{\sqrt{3}}{2} + \frac{1}{2}\times\frac{\sqrt{3}}{2}) - 0 = \pi + \frac{9\sqrt{3}}{4}$	M1 A1	
		shaded area $= 2(\pi + \frac{9\sqrt{3}}{4}) - \frac{9\sqrt{3}}{4} = 2\pi + \frac{9\sqrt{3}}{4}$	M1 A1	**(17)**
			Total	**(75)**

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Performance Record – P4 Paper G

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	inequality	series	numerical methods	complex nos.	2^{nd} order diff. eqn	1^{st} order diff. eqn	polar coords	
Marks	7	7	9	11	12	12	17	75
Student								

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