GCE Examinations

Pure Mathematics
Module P4

Advanced Subsidiary / Advanced Level

Paper F

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

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1.

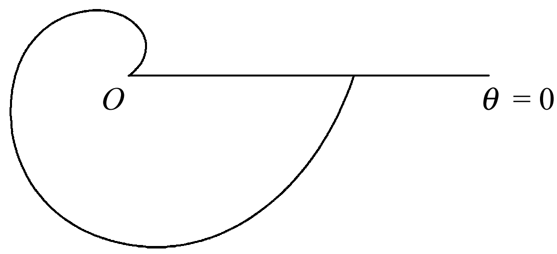

Fig. 1

Figure 1 shows the curve with polar equation

$$r = a\theta, \quad 0 \le \theta < 2\pi, \quad a > 0.$$

Find the area of the finite region bounded by the curve and the initial line $\theta = 0$. **(4 marks)**

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2. Find the set of values of x for which

$$\frac{(x-1)(x+2)}{x+4} > 4.$$ **(7 marks)**

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3. $$f(x) = 3x^5 - 7x^2 + 3.$$

(a) Show that there is a root, α, of the equation $f(x) = 0$ in the interval [0, 1]. **(2 marks)**

(b) Use linear interpolation once on the interval [0, 1] to estimate the value of α. **(2 marks)**

There is another root, β, of the equation $f(x) = 0$ close to -0.62

(c) Use the Newton-Raphson method once to obtain a second approximation to β, giving your answer correct to 3 decimal places. **(3 marks)**

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4. The Cartesian equation of the curve C is

$$(x^2 + y^2)^2 = a^2(x^2 - y^2).$$

(a) Show that, in polar coordinates, the equation of curve C can be written as

$$r^2 = a^2\cos 2\theta.$$ **(4 marks)**

(b) Sketch the curve C for $0 \le \theta < 2\pi$. **(3 marks)**

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5. *(a)* Show that the substitution $y = \dfrac{1}{u}$ transforms the differential equation

$$\frac{dy}{dx} + \frac{y}{x} - xy^2 = 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{du}{dx} - \frac{u}{x} + x = 0.$$

(3 marks)

(b) Hence find the solution of differential equation (I) such that $y = 1$ when $x = 1$, giving your answer in the form $y = f(x)$.

(7 marks)

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6. *(a)* Find $\sum_{r=n+1}^{2n} r^2$ in terms of n. **(4 marks)**

(b) Hence, or otherwise, show that

$$4 \leq \frac{\sum_{r=n+1}^{2n} r^2}{\sum_{r=1}^{n} r^2} < 7$$

for all positive integer values of n. **(6 marks)**

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7. A particle moves along the x-axis such that at time t its x-coordinate satisfies the differential equation

$$2\frac{d^2x}{dt^2} - 5\frac{dx}{dt} - 3x = 20 \sin t.$$

(a) Find the general solution of this differential equation. **(10 marks)**

Initially the particle is at $x = 5$.

Given that the particle's x-coordinate remains finite as $t \to \infty$,

(b) find an expression for x in terms of t. **(4 marks)**

Turn over

8. The complex numbers z_1 and z_2 are given by

$$z_1 = \frac{1+\mathrm{i}}{1-\mathrm{i}}, \qquad z_2 = \frac{\sqrt{2}}{1-\mathrm{i}}.$$

(a) Find z_1 in the form $a + \mathrm{i}b$ where a and b are real. **(2 marks)**

(b) Write down the modulus and argument of z_1. **(2 marks)**

(c) Find the modulus and argument of z_2. **(4 marks)**

(d) Show the points representing z_1, z_2 and $z_1 + z_2$ on the same Argand diagram, and hence find the exact value of $\tan \frac{3\pi}{8}$.

(8 marks)

END