

# GCE Examinations

# Pure Mathematics

# Module P4

Advanced Subsidiary / Advanced Level

## Paper C

Time: 1 hour 30 minutes

### *Instructions and Information*

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Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.  
Answers without working will gain no credit.



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1. Find the set of values of  $x$  for which

$$|x - 2| > 2|x + 1|. \quad \text{(6 marks)}$$

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2. (a) By using the substitution  $y = vx$ , or otherwise, find the general solution of the differential equation

$$xy \frac{dy}{dx} = x^2 + y^2. \quad \text{(7 marks)}$$

- (b) Given also that  $y = 2$  when  $x = 1$ , show that for  $x > 0$

$$y^2 = 2x^2(\ln x + 2). \quad \text{(2 marks)}$$

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3. (a) Find the sum of the series

$$2^3 + 4^3 + 6^3 + \dots + (2n)^3,$$

giving your answer in a simplified form. (3 marks)

- (b) Hence, or otherwise, show that the sum of the series

$$1^3 - 2^3 + 3^3 - 4^3 + \dots + (2n - 1)^3 - (2n)^3$$

is  $-n^2(4n + 3)$ . (6 marks)

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4. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 2e^{3x}. \quad \text{(10 marks)}$$

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5.

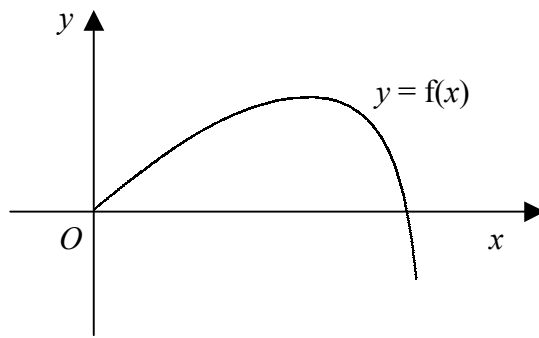


Fig. 1

Figure 1 shows part of the curve  $y = f(x)$  where

$$f(x) \equiv 2x - \tan x, \quad x \in \mathbb{R}, \quad 0 \leq x < \frac{\pi}{2}.$$

- (a) Show that there is a root,  $\alpha$ , of the equation  $f(x) = 0$  in the interval  $(1, 1.5)$ . **(2 marks)**
- (b) Use the Newton-Raphson method with an initial value of  $x = 1.25$  to find  $\alpha$  correct to 2 decimal places and justify the accuracy of your answer. **(7 marks)**
- (c) Explain with the aid of a diagram why the Newton-Raphson method fails if an initial value of  $x = 0.75$  is used when trying to find  $\alpha$ . **(3 marks)**
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6. The complex numbers  $z$  and  $w$  are defined such that

$$\begin{aligned} 3z + w &= 14, \text{ and} \\ z - iw &= 15 - 9i. \end{aligned}$$

- (a) Show that  $z = 3 - 4i$  and find  $w$  in the form  $a + ib$ , where  $a$  and  $b$  are real numbers. **(6 marks)**
- (b) Find the square roots of  $z$  in the form  $c + id$ , where  $c$  and  $d$  are real numbers. **(7 marks)**
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*Turn over*

7.

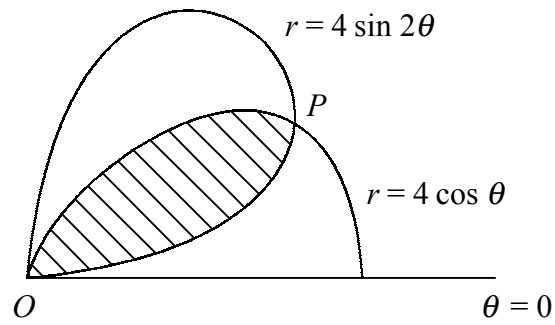


Fig. 2

Figure 2 shows the curves with polar equations

$$r = 4 \sin 2\theta \quad 0 \leq \theta \leq \frac{\pi}{2},$$

$$r = 4 \cos \theta \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

- (a) Find the polar coordinates of the point  $P$  where the two curves intersect. **(5 marks)**
- (b) Find the exact area of the shaded region bounded by the two curves. **(11 marks)**
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**END**