GCE Examinations

Pure Mathematics Module P4

Advanced Subsidiary / Advanced Level

Paper G

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. Find the set of values of x for which

$$\frac{x^2 - 12}{x} \geq 1.$$ **(7 marks)**

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2. Show that the sum of the first n terms of the series

$$5^2 + 9^2 + 13^2 + 17^2 + \ldots$$

is given by $\frac{1}{3}n(16n^2 + 36n + 23)$. **(7 marks)**

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3. $$\text{f}(x) \equiv x^3 - 5x^2 + 2.$$

(a) Show that the equation $\text{f}(x) = 0$ has a root α in the interval $[0, 1]$. **(2 marks)**

(b) Use the Newton-Raphson method with initial value $x = 0.5$ to find a value for α which is correct to 2 decimal places.
(5 marks)

(c) Give a reason why the Newton-Raphson method fails if an initial value of $x = 0$ is used in part *(b)*.
(2 marks)

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4. The complex number z is given by

$$z = \frac{1 + \text{i}\sqrt{3}}{1 - \text{i}\sqrt{3}}.$$

(a) Show that z can be expressed in the form

$$\lambda(1 - \text{i}\sqrt{3})$$

where λ is a rational number which you should find. **(4 marks)**

(b) Find the modulus and argument of z. **(3 marks)**

(c) Hence, or otherwise, find the modulus and argument of

$$\left(\frac{1 + \text{i}\sqrt{3}}{1 - \text{i}\sqrt{3}}\right)^4.$$ **(4 marks)**

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5. *(a)* Find the values of p and q such that $y = p\sin x + q\cos x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = \sin x.$$ **(7 marks)**

(b) Find the general solution of this differential equation. **(5 marks)**

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6. *(a)* Show that

$$\int 2\cot x \; dx = \ln(\sin^2 x) + c,$$

where c is an arbitrary constant. **(3 marks)**

(b) Find the general solution of the differential equation

$$\sin x\frac{dy}{dx} + 2y\cos x = 1.$$ **(5 marks)**

Given that $y = 0$ when $x = \frac{\pi}{4}$,

(c) show that when $x = \frac{\pi}{3}$,

$$y = \tfrac{2}{3}(\sqrt{2} - 1).$$ **(4 marks)**

Turn over

7.

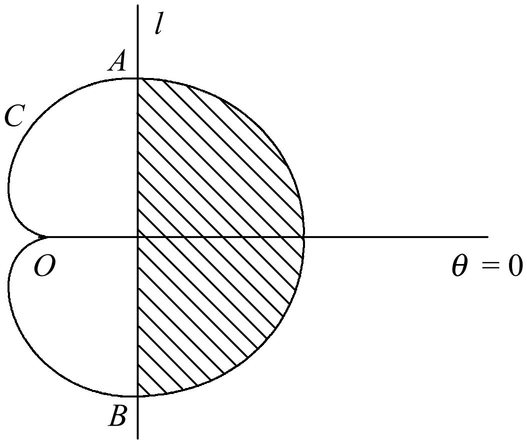

Fig. 1

Figure 1 shows the curve C with polar equation

$$r = 2(1 + \cos\theta), \quad -\pi < \theta \leq \pi,$$

and the line l with polar equation

$$r\cos\theta = \tfrac{3}{2},$$

referred to the pole O and initial line $\theta = 0$.

(a) Find the polar coordinates of the points A and B, where l intersects C. **(6 marks)**

(b) Show that the area of triangle OAB is $\dfrac{9\sqrt{3}}{4}$. **(3 marks)**

(c) Hence find the area of the shaded region bounded by C and l. **(8 marks)**

END