GCE Examinations

Advanced Subsidiary / Advanced Level

# Pure Mathematics
# Module P4

Paper F

# MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

# P4 Paper F – Marking Guide

**1.**  area $= \frac{1}{2} \int_0^{2\pi} a^2\theta^2 \; d\theta$    M1

$\qquad = \frac{1}{2}a^2 [\frac{1}{3}\theta^3]_0^{2\pi}$    A1

giving $\frac{4}{3}a^2\pi^3$    M1 A1  **(4)**
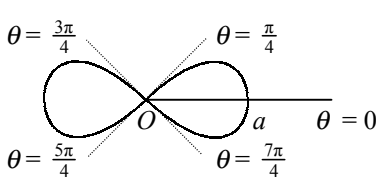
---

**2.**  $\frac{(x-1)(x+2)}{x+4} - 4 > 0 \; \therefore \; \frac{x^2-3x-18}{x+4} > 0$    M1 A1

$\frac{(x+3)(x-6)}{x+4} > 0 \; \therefore \;$ critical values are $-4, -3, 6$    M1 A1

considering change of sign of factors gives
$-4 < x < -3 \;$ or $\; x > 6$    M1 A2  **(7)**

---

**3.**  *(a)*  f(0) = 3; f(1) = −1;
f cont. over interval, change of sign $\therefore$ root    M1 A1

*(b)*  $\alpha \approx 0 + \frac{3}{3+1} \times 1 = 0.75$    M1 A1

*(c)*  f$'(x) = 15x^4 - 14x, \; x_{n+1} = x_n - \dfrac{3x_n^{\,5} - 7x_n^{\,2} + 3}{15x_n^{\,4} - 14x_n}$    M1 A1

giving $\beta \approx -0.623$ (3dp)    A1  **(7)**

---

**4.**  *(a)*  $x = r\cos\theta, \; y = r\sin\theta$
$\therefore \; (r^2\cos^2\theta + r^2\sin^2\theta)^2 = a^2(r^2\cos^2\theta - r^2\sin^2\theta)$    M1 A1
$[r^2(\cos^2\theta + \sin^2\theta)]^2 = a^2r^2(\cos^2\theta - \sin^2\theta)$    M1
$r^4 = a^2r^2\cos 2\theta \;$ giving $\; r^2 = a^2\cos 2\theta$    A1

*(b)*



B3  **(7)**

---

**5.**  *(a)*  $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = -\dfrac{1}{u^2}\dfrac{du}{dx}$    M1

$\therefore \; -\dfrac{1}{u^2}\dfrac{du}{dx} + \dfrac{1}{xu} - \dfrac{x}{u^2} = 0$    M1

$\dfrac{du}{dx} - \dfrac{u}{x} + x = 0$    A1

*(b)*  int. fac. $= e^{\int -\frac{1}{x} dx} = e^{-\ln|x|} = \dfrac{1}{x}$    M1 A1

$\therefore \; \dfrac{1}{x}\dfrac{du}{dx} - \dfrac{u}{x^2} = -1$

$\dfrac{d}{dx}(\dfrac{u}{x}) = -1$    M1

$\dfrac{u}{x} = \int -1 \; dx = -x + c$    A1

$\therefore \; \dfrac{1}{xy} = -x + c$    A1

$x = 1, y = 1 \; \therefore \; c = 2$    M1

$\therefore \; \dfrac{1}{xy} = 2 - x \;$ giving $\; y = \dfrac{1}{x(2-x)}$    A1  **(10)**

---

*© Solomon Press*

**6.** *(a)* $\displaystyle\sum_{r=n+1}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2$      M1

$= \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}n(n+1)(2n+1)$      A1

$= \frac{1}{6}n(2n+1)[2(4n+1) - (n+1)]$      M1

$= \frac{1}{6}n(2n+1)(7n+1)$      A1

*(b)* $\text{LHS} = \dfrac{\frac{1}{6}n(2n+1)(7n+1)}{\frac{1}{6}n(n+1)(2n+1)} = \dfrac{7n+1}{n+1}$      M1 A1

$= \dfrac{7n+7-6}{n+1} = 7 - \dfrac{6}{n+1}$      M1

$n \geq 1; \ n+1 \geq 2 \ \therefore 0 < \dfrac{6}{n+1} \leq 3 \ \therefore 4 \leq 7 - \dfrac{6}{n+1} < 7$ as required      M1 A2   **(10)**
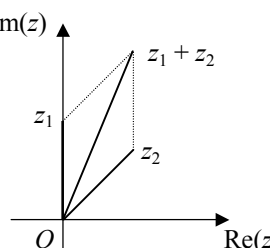
---

**7.** *(a)* aux. eqn. $2m^2 - 5m - 3 = 0$      M1

$(2m+1)(m-3) = 0; \ m = -\frac{1}{2}, 3$   C.F. $x = A\,e^{-\frac{1}{2}t} + B\,e^{3t}$      A1

for P.I. try $x = p \sin t + q \cos t$      M1

$\dfrac{dx}{dt} = p \cos t - q \sin t, \quad \dfrac{d^2x}{dt^2} = -p \sin t - q \cos t$      M1 A1

$-2p\sin t - 2q\cos t - 5p\cos t + 5q\sin t - 3p\sin t - 3q\cos t = 20\sin t$      M1

$-5p + 5q = 20$

$-5p - 5q = 0$      A1

giving $p = -2, \ q = 2$      M1 A1

$\therefore \ x = A\,e^{-\frac{1}{2}t} + B\,e^{3t} - 2\sin t + 2\cos t$      A1

*(b)* $x$ finite as $t \to \infty \ \therefore B = 0$      M1 A1

$t = 0, x = 5 \ \therefore \ 5 = A - 0 + 2$   giving $A = 3$      M1

$\therefore \ x = 3\,e^{-\frac{1}{2}t} - 2\sin t + 2\cos t$      A1   **(14)**

---

**8.** *(a)* $z_1 = \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{2i}{2} = i$      M1 A1

*(b)* $\text{mod } z_1 = 1; \ \arg z_1 = \frac{\pi}{2}$      B2

*(c)* $z_2 = \dfrac{\sqrt{2}}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{\sqrt{2}}{2}(1+i)$      M1 A1

$\text{mod } z_2 = \dfrac{\sqrt{2}}{2} \times \sqrt{2} = 1$      A1

$\arg z_2 = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$      A1

*(d)*

     B3

$\arg(z_1 + z_2) = \frac{1}{2}(\frac{\pi}{2} + \frac{\pi}{4}) = \frac{3\pi}{8}$      M1 A1

$z_1 + z_2 = i + \dfrac{\sqrt{2}}{2}(1+i) = \dfrac{\sqrt{2}}{2} + (1 + \dfrac{\sqrt{2}}{2})i$      B1

$\therefore \ \tan\dfrac{3\pi}{8} = \dfrac{1 + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \dfrac{2 + \sqrt{2}}{\sqrt{2}} = \sqrt{2} + 1$      M1 A1   **(16)**

---

Total   **(75)**

*© Solomon Press*

# Performance Record – P4 Paper F

*© Solomon Press*

| Question no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
|---|---|---|---|---|---|---|---|---|---|
| Topic(s) | polar coords | inequality | numerical methods | polar coords | 1st order diff. eqn | series | 2nd order diff. eqn | complex nos. | |
| Marks | 4 | 7 | 7 | 7 | 10 | 10 | 14 | 16 | 75 |
| Student | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

*© Solomon Press*