

GCE Examinations
Advanced Subsidiary / Advanced Level
Pure Mathematics
Module P4

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



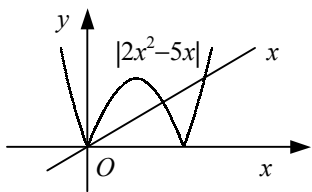
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P4 Paper B – Marking Guide

1.



considering $2x^2 - 5x = x$ gives $2x(x - 3) = 0 \therefore x = 0, 3$
 considering $-(2x^2 - 5x) = x$ gives $2x(x - 2) = 0 \therefore x = 0, 2$
 using graphs, require $2 < x < 3$

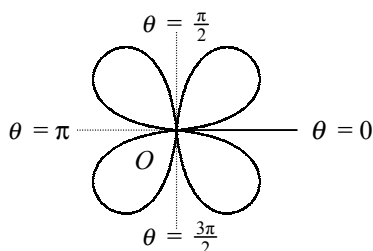
B2

M1 A1

A1

A1 (6)

2. (a)



B3

$$(b) \quad \frac{1}{2} \int_0^{\frac{\pi}{2}} a^2 \sin^2 2\theta \, d\theta$$

M1

$$= \frac{1}{4} a^2 \int_0^{\frac{\pi}{2}} 1 - \cos 4\theta \, d\theta$$

M1

$$= \frac{1}{4} a^2 \left[\theta - \frac{1}{4} \sin 4\theta \right]_0^{\frac{\pi}{2}}$$

A1

giving $\frac{1}{8} a^2 \pi$

M1 A1 (8)

3. (a)

$$= \sum_{r=1}^n (r^3 - r^2 + r - 1)$$

M1

$$= \frac{1}{4} n^2(n+1)^2 - \frac{1}{6} n(n+1)(2n+1) + \frac{1}{2} n(n+1) - n$$

M1 A1

$$= \frac{1}{12} n[3n(n^2 + 2n + 1) - 2(2n^2 + 3n + 1) + 6(n+1) - 12]$$

M1

$$= \frac{1}{12} n[3n^3 + 2n^2 + 3n - 8]$$

A1

$$= \frac{1}{12} n(n-1)(3n^2 + 5n + 8)$$

A1

$$(b) \quad \sum_{r=5}^{25} (r^2 + 1)(r - 1) = \sum_{r=1}^{25} (r^2 + 1)(r - 1) - \sum_{r=1}^4 (r^2 + 1)(r - 1)$$

M1

$$= \frac{1}{12} \cdot 25 \cdot 24 \cdot 2008 - \frac{1}{12} \cdot 4 \cdot 3 \cdot 76$$

M1

$$= 100\,400 - 76 = 100\,324$$

A1 (9)

4. (a)

$$\text{int. fac.} = e^{\int -\cot x \, dx} = e^{-\ln|\sin x|} = \text{cosec } x$$

M1 A1

$$\therefore \text{cosec } x \frac{dy}{dx} - y \text{cosec } x \cot x = \text{cosec } x \times 2 \sin x \cos x$$

M1

$$\frac{d}{dx} (y \text{cosec } x) = 2 \cos x$$

A1

$$y \text{cosec } x = \int 2 \cos x \, dx$$

$$y \text{cosec } x = 2 \sin x + c \quad \text{or} \quad y = 2 \sin^2 x + c \sin x$$

M1 A1

$$(b) \quad x = \frac{\pi}{6}, y = 2 \therefore c = 3$$

M1 A1

$$\therefore \text{when } x = \frac{2\pi}{3}, y = \frac{3}{2}(1 + \sqrt{3})$$

A1 (9)

5. (a) $f(1.0) = -0.0986$; $f(1.1) = 0.305$
 f cont. over interval, change of sign \therefore root M1 A1
- (b) $f'(x) = 3x^2 + \frac{2x}{4-x^2}$ M1 A1
- $$x_{n+1} = x_n - \frac{x_n^3 - \ln(4-x_n^2)}{3x_n^2 + \frac{2x_n}{4-x_n^2}}$$
- $x_0 = 1.0, x_1 = 1.026894, x_2 = 1.026221$ A2
- $f(1.0262205) = -2.02 \times 10^{-6}$; $f(1.0262215) = 1.83 \times 10^{-6}$
change of sign \therefore root $\therefore \alpha = 1.026221$ correct to 6dp M1 A1 (10)

6. (a) $|z_1| = \sqrt{(49+1)} = \sqrt{50} = 5\sqrt{2}$ B1
 $5\sqrt{2} \times |z_3| = 50 \therefore |z_3| = 5\sqrt{2}$ M1 A1
- (b) $\arg z_2 = \tan^{-1} \sqrt{3} = \frac{\pi}{3}$ B1
 $\frac{\pi}{3} - \arg z_3 = \frac{7\pi}{12} \therefore \arg z_3 = -\frac{\pi}{4}$ M1 A1
- (c) $z_3 = 5\sqrt{2}[\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4})] = 5 - 5i \therefore a = 5, b = -5$ M1 A1
- (d) $\frac{z_1}{z_3} = \frac{7-i}{5-5i} \times \frac{5+5i}{5+5i} = \frac{40+30i}{50} = \frac{1}{5}(4+3i)$ M2 A1
- (e) B2
- (f) e.g. $|z_1| = |z_3| \therefore |\frac{z_1}{z_3}| = |\frac{z_3}{z_1}|$; $\arg \frac{z_1}{z_3} = \arg z_1 - \arg z_3 = -\arg \frac{z_3}{z_1}$
same modulus, -ve of argument \therefore conjugate M1 A1 (15)

7. (a) $\frac{dx}{dt} = e^t$; $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = e^{-t} \frac{dy}{dt}$ M1 A1
- $$\frac{d^2y}{dx^2} = \frac{dt}{dx} (e^{-t} \frac{d^2y}{dt^2} - e^{-t} \frac{dy}{dt}) = e^{-2t} (\frac{d^2y}{dt^2} - \frac{dy}{dt})$$
- M1 A2
- (b) $e^{2t} \cdot e^{-2t} (\frac{d^2y}{dt^2} - \frac{dy}{dt}) - e^t \cdot e^{-t} \cdot \frac{dy}{dt} - 3y = 6e^{2t}$ M1 A1
- giving $\frac{d^2y}{dt^2} - 2 \frac{dy}{dt} - 3y = 6e^{2t}$ A1
- (c) aux. eqn. $m^2 - 2m - 3 = 0$ M1
 $(m+1)(m-3) = 0$; $m = -1, 3$ C.F. $y = Ae^{-t} + Be^{3t}$ A1
- for P.I. try $y = Ce^{2t} \therefore \frac{dy}{dt} = 2Ce^{2t}$, $\frac{d^2y}{dt^2} = 4Ce^{2t}$ M1
- so $4Ce^{2t} - 4Ce^{2t} - 3Ce^{2t} = 6e^{2t} \therefore C = -2$ M1 A1
- gen. soln. $y = Ae^{-t} + Be^{3t} - 2e^{2t} = \frac{A}{x} + Bx^3 - 2x^2$ A1
- $$\frac{dy}{dx} = -\frac{A}{x^2} + 3Bx^2 - 4x$$
- M1
- $x = 1, y = 3 \therefore 3 = A + B - 2$
- $x = 1, \frac{dy}{dx} = -5 \therefore -5 = -A + 3B - 4$ giving $A = 4, B = 1$ M1 A1
- $\therefore y = x^3 - 2x^2 + \frac{4}{x}$ A1 (18)

Total (75)

