1. Find the modulus and argument of the complex number $\frac{-3+\mathrm{i}}{1-2\mathrm{i}}$. **(4 marks)**

2. Prove that $\sum_{r=0}^{n}(r^3+r-1)=\frac{1}{4}(n^2-1)((n+1)^2+3)$. **(5 marks)**

3. (a) Sketch the curve C with polar equation $r = a(1 + 2\sin\theta)$, $0 \le \theta \le 2\pi$, where $a > 0$.
   Show clearly how the curve behaves close to the pole. **(4 marks)**

   (b) On the same diagram, sketch the circle with polar equation $r = 2a\sin\theta$ and state the polar coordinates of its centre. **(2 marks)**

4. $\mathrm{f}(x) \equiv x^2 - \mathrm{e}^x - 1$.

   (a) Show that there is a root of the equation $\mathrm{f}(x) = 0$ in the interval $(-2,-1)$. **(2 marks)**

   (b) Use the interval bisection process three times to find an estimate x_0 for this root. **(4 marks)**

   (c) Find the set of values of x for which $\mathrm{f}(x) > 1 - \mathrm{e}^x$. **(3 marks)**

5. (a) Obtain the general solution of the differential equation
$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2x + 3t.$$
(7 marks)

   (b) Given that $x = 1{\cdot}25$ when $t = 0$, find the value of x, to 3 significant figures, when $t = 1{\cdot}5$. **(4 marks)**

6. The complex numbers w and z are given by
$$w = 3 + 4\mathrm{i},\quad z = \frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}.$$

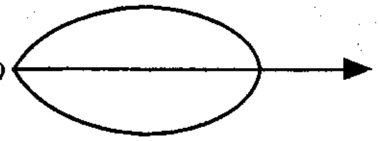
   (a) Show on a single Argand diagram the points P and Q which represent w and z respectively. **(2 marks)**

   (b) Verify that ww^* is real. **(2 marks)**

   (c) Find the modulus and the argument (in radians, to 2 decimal places if necessary) of w and of z. **(4 marks)**

   (d) Express wz in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where θ is given in radians to 2 decimal places. **(4 marks)**

7. The diagram shows the curve with polar equation $r = 4\cos 3\theta$, for $-\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}$.



(a) Find, in terms of π, the area of the region enclosed by the curve. **(6 marks)**

(b) Show that, at the points on the curve at which the tangents are parallel to the initial line, $3\tan\theta\tan 3\theta = 1$. **(6 marks)**

8. (a) Find the general solutions of the differential equations

(i) $2\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 17y = 0,$ **(5 marks)**

(ii) $2\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 17y = 17x + 1.$ **(5 marks)**

(b) Given that $y = 1$ when $x = 0$ and $\frac{dy}{dx} = -1$ when $x = 0$, find the particular solution of the equation in part (a) (ii). **(6 marks)**