1. Given that $z = a + (1 - a)\mathrm{i}$ and that $zz^* = \frac{5}{8}$, find the two possible values of the real number a. **(4 marks)**

2. (a) Sketch, on one diagram, the lines with polar equation $\theta = \frac{2\pi}{3}$ and $r = \sec(\pi - \theta)$. **(3 marks)**

   (b) Find the polar coordinates of the point of intersection of these two lines. **(3 marks)**

3. Using standard results for the summation of series, prove that

$$\sum_{r=1}^{n} (3r + 1)(3r - 2) = n(3n^2 + 3n - 2).$$ **(7 marks)**

4. (a) Sketch on the same diagram the curves with equations

$$y = \frac{1}{x+1} \quad \text{and} \quad y = \frac{x}{x-1}.$$ **(4 marks)**

   (b) Using your sketch, or otherwise, find the solution set of the inequality

$$\frac{1}{x+1} \geq \frac{x}{x-1}.$$ **(3 marks)**

5. Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 9y = \sin x + \cos x.$$ **(8 marks)**

6. (a) Using the same axes, sketch the graphs of $y = \ln(x + 1)$ and $y = 2\cos 3x$ for $0 \leq x \leq \pi$. Show the coordinates of any points where the graphs cross the axes. **(4 marks)**

   (b) Given that the smallest positive value of x at which the graphs intersect is α, show that $0 < \alpha < 1$. **(2 marks)**

   (c) Taking $\frac{\pi}{6}$ as a first approximation to α, use the Newton-Raphson process twice to obtain a better approximation, correct to 4 significant figures. **(6 marks)**

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7. (a) Show that the substitution $y = \frac{1}{z}$ transforms the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = x^3 y^2$$

into the equation $\frac{dz}{dx} - \frac{z}{x} = -x^3$. **(5 marks)**

(b) Hence find y in terms of x, given that $y = 1$ when $x = 1$. **(8 marks)**

8. The cubic equation $x^3 + bx^2 + cx + d = 0$ has roots $z_1 = 1$, $z_2 = 2 - i$ and $z_3 = m + ni$.

(a) State the values of m and n. **(2 marks)**

(b) Find the values of the real constants b, c and d. **(5 marks)**

(c) Show on an Argand diagram the points representing the three roots. **(3 marks)**

(d) For each of the three roots, find

(i) the modulus,

(ii) the argument, in radians to 3 significant figures. **(5 marks)**

(e) Find, in the form $p + qi$, the complex number $\frac{z_2}{z_3}$. **(3 marks)**

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