1. Given that $\sum_{r=1}^{n} 7r = \sum_{r=1}^{n} r^2$, find the value of the positive integer n. **(4 marks)**

2. One root of the equation $x^3 + 4x^2 + 2x + k = 0$ is $\mathrm{i}\sqrt{2}$.

   Find the value of the real number k and solve the equation completely. **(6 marks)**

3. (a) If $u_r = 2^r - 1$, show that $u_r - u_{r-1} = 2^{r-1}$. **(3 marks)**

   (b) Deduce, without using standard results for geometric series, that $\sum_{r=1}^{n} 2^{r-1} = 2^n - 1$. **(4 marks)**

4. $\mathrm{f}(x) \equiv 5 - 2x - \dfrac{2}{x^2}$. The equation $\mathrm{f}(x) = 0$ has a root α in the interval $(-1, 0)$, a root β in the interval $(0, 1)$ and a root γ in the interval $(2, 3)$.

   (a) Express in terms of α, β and γ the solution set of the inequality $\mathrm{f}(x) \geq 0$. **(4 marks)**

   (b) Use linear interpolation once on the interval $(2, 3)$ to find an estimate of γ, correct to 3 significant figures. **(3 marks)**

5. Given that $\sin x \dfrac{\mathrm{d}y}{\mathrm{d}x} - y\cos x = 2$, where $0 < x < \pi$,

   and that $y = 0$ when $x = \dfrac{\pi}{4}$,

   (a) express y in terms of x; **(8 marks)**

   (b) find the value of y when $x = \dfrac{\pi}{6}$, giving your answer in surd form. **(2 marks)**

6. The complex number z is defined by $z = \dfrac{5\mathrm{i}}{\mathrm{i} - 3}$.

   (a) Show that $|z| = \dfrac{1}{2}\sqrt{10}$, and find $\arg z$ in radians to 2 decimal places. **(4 marks)**

   The points P and Q in the Argand diagram represent the complex numbers z and $\mathrm{i}z$ respectively. O is the origin.

   (b) State the size of angle POQ. **(1 mark)**

   (c) Find the complex number w represented by the point R, where OR is equal and parallel to PQ. **(3 marks)**

   (d) Find $|w|$ and interpret your answer as a length in the Argand diagram. **(3 marks)**

7. The curves C_1 and C_2 have polar equations $r = a(1 + \sin \theta)$ and $r = 2a \sin \theta$, where $a > 0$ and $0 \leq \theta < 2\pi$.

   (a) Sketch C_1 and C_2 and state the polar coordinates of the point, other than the pole, at which they meet. **(6 marks)**

   (b) Calculate, in terms of π, the area of the finite region which lies inside C_1 but outside C_2. **(9 marks)**

8. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 10e^{3x},$$

   given that when $x = 0$, $y = -2$ and $\frac{dy}{dx} = 2$. **(13 marks)**

   (b) Show that, when $x = \frac{\pi}{2}$, $y = 315$ to the nearest integer. **(2 marks)**