1. Given that the equation $x^3 + x + 1 = 0$ has a root α in the interval $(-0{\cdot}8, -0{\cdot}5)$, use linear interpolation once over this interval to find an estimate of α to 2 decimal places. **(5 marks)**

2. Find the value of θ at the point on the curve with polar equation

$$r = ae^{\theta},\ a > 0,\ 0 \le \theta < \pi,$$

at which the tangent is perpendicular to the initial line. **(6 marks)**

3. Solve the inequality

$$|1 - x| < x^2 - 1.$$ **(7 marks)**

4. (a) Prove that $\sum_{r=1}^{n} 2r(r-3) = \frac{2}{3}n(n+1)(n-4)$. **(5 marks)**

(b) Hence find $\sum_{r=10}^{30} 2r(r-3)$. **(3 marks)**

5. Solve the differential equation

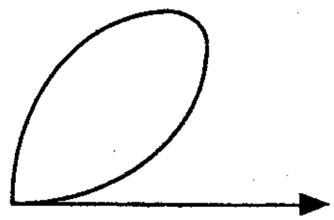
$$\frac{dy}{dx} - \frac{y}{x} = \ln 2x,$$

where $x > 0$, given that $y = 1$ when $x = \frac{1}{2}$. **(9 marks)**

6. The diagram shows the curve whose cartesian equation is

$$(x^2 + y^2)^2 = 2a^2xy,\ x > 0, y > 0,$$

where a is a positive constant.



(a) Show that the polar equation of the curve is $r^2 = a^2 \sin 2\theta$. **(3 marks)**

(b) Find the area of the region contained by the curve. **(5 marks)**

The straight line $r = a\sec(\alpha - \theta)$ touches the curve at the point with polar coordinates $\left(\frac{\pi}{4}, 1\right)$.

(c) Find the value of α. **(3 marks)**

7. The complex number z is given by

$$z = \frac{a + b\mathrm{i}}{3 + 4\mathrm{i}}$$

where a and b are integers.

(a) Express $|z|$ in terms of a and b. **(3 marks)**

(b) Given that $\arg z = \frac{3\pi}{4}$, show that $a = 7b$. **(5 marks)**

(c) Given also that $|z| = \sqrt{2}$, find the values of a and b. **(4 marks)**

(d) Show on an Argand diagram the points representing z and z^*. **(2 marks)**

8. (a) Show that the substitution $y = ue^x$, where u is a function of x, transforms the differential equation

$$(2x + 1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (3 - 4x)\frac{\mathrm{d}y}{\mathrm{d}x} + 2(x - 2)y = 0, \; x > 0,$$

into the equation

$$(2x + 1)\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 5\frac{\mathrm{d}u}{\mathrm{d}x} = 0.$$ **(7 marks)**

(b) By means of the further substitution $v = \frac{\mathrm{d}u}{\mathrm{d}x}$, solve this equation to obtain v in terms of x. **(5 marks)**

(c) Hence find y in terms of x and two arbitrary constants. **(3 marks)**