1. Given that $S_n = \sum_{r=1}^{n} (r^3 + r)$, show that $S_n = an(n+1)(n^2 + n + b)$,
where a and b are rational numbers to be found **(5 marks)**

2. Given that $w = -1 + 5\text{i}$ and $z = 4 - 2\text{i}$, find the complex number $\frac{w^2}{z}$ in the form $p + q\text{i}$, where p and q are rational numbers to be found. **(6 marks)**

3. Find the general solution of the differential equation
$$3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 12y = 0.$$ **(6 marks)**

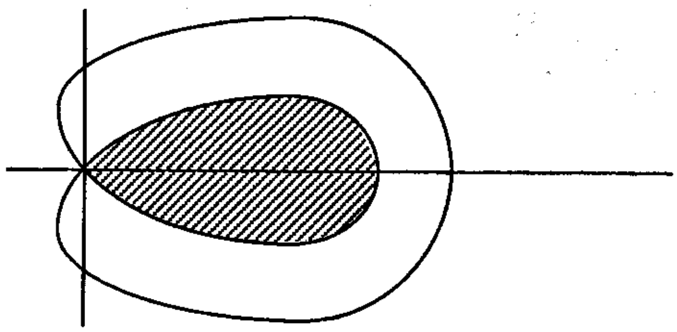
4. The functions f and g are defined by
$$f : x \to \frac{2x}{x+1},\ x \in \mathbb{R}, x \neq -1,$$
$$g : x \to |x|,\ x \in \mathbb{R}.$$
Find the set of values of x for which $f(x) < g(x)$. **(7 marks)**

5. (a) Show that the equation $\sec x - 3x = 0$ has a root in the interval (1, 1·4). **(2 marks)**
   (b) Taking 1·3 as a first approximation to this root, carry out two applications of the Newton-Raphson procedure to obtain a better approximation to the root. Give your answer to 3 decimal places. **(6 marks)**

6. The points P, Q and R in the Argand diagram represent the complex numbers $z_1 = 5 - 5\text{i}$, $z_2 = -7 + \text{i}$ and $z_3 = 3 + 4\text{i}$ respectively.
   (a) Calculate the modulus and the argument (in radians to 2 decimal places) of each of z_1, z_2 and z_3. **(6 marks)**
   (b) Calculate the perimeter of triangle PQR. **(5 marks)**
   (c) Describe the transformation which maps ΔPQR onto $\Delta P'Q'R'$, where the points P', Q' and R' represent the complex numbers z_1^*, z_2^* and z_3^*. **(2 marks)**

7. The diagram shows the curve C with polar equation

$$r = 6\cos\theta - 1, \quad 0 \le \theta < 2\pi.$$



Calculate

(a) the values of r at the four points on C at which the tangents are parallel to the initial line, **(7 marks)**

(b) the shaded area contained by the loop of the curve. **(8 marks)**

8. (a) Show that the substitution $v = y^4$ transforms the differential equation

$$\frac{dy}{dx} - \frac{y}{2x} = \frac{5x^2}{y^3}, \quad x > 0,\ y > 0,$$

into an equation of the form $\dfrac{dv}{dx} + P(x)v = Q(x)$,

where $P(x)$ and $Q(x)$ are functions of x to be found. **(5 marks)**

(b) By finding an integrating factor for the transformed equation, obtain the general solution of the original differential equation in the form $y = f(x)$. **(6 marks)**

(c) Given further that $y = 2$ when $x = 1$, state the smallest value of x for which y is real. **(4 marks)**