PURE MATHS 4 (A) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1. $\frac{7}{2}n(n+1) = \frac{1}{6}n(n+1)(2n+1)$ $\quad 2n+1 = 21 \quad n = 10$ — M1 A1 M1 A1 — **4**

2. Substitute $x = \mathrm{i}\sqrt{2}$: $2\mathrm{i}\sqrt{2} - 8 + 2\mathrm{i}\sqrt{2} + k = 0$ $\quad k = 8$ — M1 A1
   Another root is $-\mathrm{i}\sqrt{2}$ $\quad (x - \mathrm{i}\sqrt{2})(x + \mathrm{i}\sqrt{2}) = x^2 + 2$ — B1 M1 A1
   Dividing, other factor is $x + 4$, so third root is -4 — A1 — **6**

3. (a) $u_r - u_{r-1} = 2^r - 1 - 2^{r-1} + 1 = 2^r - 2^{r-1} = 2^{r-1}(2-1) = 2^{r-1}$ — M1 A1 A1
   (b) $\Sigma\, 2^{r-1} = \Sigma\,(u_r - u_{r-1}) = u_n - u_0 = (2^n - 1) - 0 = 2^n - 1$ — M1 A1 M1 A1 — **7**

4. (a) From a sketch or otherwise, solution set is $x \le \alpha,\ \beta \le x \le \gamma$ — M1 M1 A1 A1
   (b) $2 + (1/2)/(1/2 + 11/9) \times 1 = 2{\cdot}29$ — M1 A1 A1 — **7**

5. $\frac{\mathrm{d}y}{\mathrm{d}x} - y\cot x = 2\operatorname{cosec} x$ $\quad$ Int. factor $= \mathrm{e}^{\int -\cot x\,\mathrm{d}x} = \mathrm{e}^{-\ln \sin x} = \operatorname{cosec} x$ — B1 M1 A1
   $\frac{1}{\sin x}\frac{\mathrm{d}y}{\mathrm{d}x} - y\frac{\cos x}{\sin^2 x} = 2\operatorname{cosec}^2 x$ $\quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{\sin x}\right) = 2\operatorname{cosec}^2 x$ — M1 A1 A1
   $y = \sin x\,(c - 2\cot x) = c\sin x - 2\cos x$ $\quad y(\pi/4) = 0 : c = 2$ — M1 A1
   When $x = \pi/6$, $y = 2(1/2 - \sqrt{3}/2) = 1 - \sqrt{3}$ — M1 A1 — **10**

6. (a) $z = 5\mathrm{i}(\mathrm{i}+3)/10 = (1-3\mathrm{i})/2$, so $|z| = (\sqrt{10})/2$ — M1 A1
   $\arg z = \arctan(-3) = -1{\cdot}25$ — M1 A1
   (b) $\mathrm{i}z = 3/2 + \mathrm{i}/2$ $\quad$ Angle $POQ = \pi/2$ — B1
   (c) $w = \mathrm{i}z - z = 1 + 2\mathrm{i}$ — M1 A1 A1
   (d) $|w| = \sqrt{5}$ $\quad$ This is the length of PQ or of OR — M1 A1 B1 — **11**

7. (a) Curves sketched : cardioid and circle. $\quad$ Meet at $(2a, \pi/2)$ — B2 B2 B2
   (b) Area inside $C_1 = \frac{1}{2}\int_0^{2\pi} a^2(1+\sin\theta)^2\,\mathrm{d}\theta$ — M1 A1
   $= \frac{a^2}{2}\int_0^{2\pi}(\sin^2\theta + 2\sin\theta + 1)\,\mathrm{d}\theta = \frac{a^2}{4}\int_0^{2\pi}(-\cos 2\theta + 4\sin\theta + 3)\,\mathrm{d}\theta$ — M1 A1 A1
   $= \frac{a^2}{4}\left[-\frac{1}{2}\sin 2\theta - 4\cos\theta + 3\theta\right]_0^{2\pi} = \frac{3\pi a^2}{2}$ — M1 A1
   Area of circle $C_2 = \pi a^2$, so area between curves $= \frac{\pi a^2}{2}$ — M1 A1 — **15**

8. (a) Aux. eqn. $u^2 - 4u + 8 = 0$ has roots $2 \pm 2\mathrm{i}$, so C.F. is — M1 A1
   $y = \mathrm{e}^{2x}(a\sin 2x + b\cos 2x)$ $\quad$ Let P.I. be $k\mathrm{e}^{3x}$ — A1 M1
   Then $9k\mathrm{e}^{3x} - 4(3k\mathrm{e}^{3x}) + 8k\mathrm{e}^{3x} = 10\mathrm{e}^{3x}$ $\quad k = 2$ — A1 A1
   General solution is $y = \mathrm{e}^{2x}(a\sin 2x + b\cos 2x) + 2\mathrm{e}^{3x}$ — A1
   $y' = \mathrm{e}^{2x}((a-2b)\sin x) + (2a+b)\cos x) + 6\mathrm{e}^{3x}$ — M1 A1
   $y(0) = -2 : b = -4$ $\quad y'(0) = 2 : 2a - 2 = 2$ $\quad a = 2$ — A1 M1 A1
   $y = 2\mathrm{e}^{2x}(\sin 2x - 2\cos 2x) + 2\mathrm{e}^{3x}$ — A1
   (b) $x = \pi/2 : y = 4\mathrm{e}^{\pi} + 2\mathrm{e}^{3\pi/2} = 315{\cdot}2$ — M1 A1 — **15**