1. Given that $z_1 = 2 + 4i$ and $z_2 = 1 - 2i$, show on an Argand diagram the points representing z_1, z_2 and z_1z_2. **(4 marks)**

2. Prove that $\sum_{r=1}^{n} (2r-1)^2 = kn(4n^2 - 1)$, where k is a constant to be found. **(6 marks)**

3. Solve the inequalities

   (i) $\dfrac{2x-1}{x+1} < 1$, (ii) $\dfrac{2x-1}{x+1} \geq 1$. **(7 marks)**

4. (a) Show that the equation $4 \ln x - 5 \ln 2x + 5 = 0$ has a root between $x = 4$ and $x = 5$. **(2 marks)**

   (b) Use the interval bisection process three times on the interval (4, 5) to find an estimate α of this root, to 3 significant figures. **(4 marks)**

   (c) Find, to 3 significant figures, the value of $4 \ln \alpha - 5 \ln 2\alpha + 5$. **(2 marks)**

5. Solve the differential equation

$$\frac{d^2y}{dx^2} + 9y = 6\frac{dy}{dx},$$

   given that when $x = 0$, $y = 3$ and $\dfrac{dy}{dx} = 2$. **(9 marks)**

6. Given that $w = a + i$ and $z = 1 + bi$, where a and b are real,

   (a) find, in terms of a and b, the real and imaginary parts of

   (i) wz, (ii) $(wz)^*$, (iii) $\dfrac{z}{w}$. **(6 marks)**

   Given further that $|w| = \sqrt{10}$ and $\left|\dfrac{z}{w}\right| = \sqrt{5}$,

   (b) find the values of the positive constants a and b. **(6 marks)**

7. (a) If $x = vt$, where v is a function of t, show that $\dfrac{dx}{dt} = v + t\dfrac{dv}{dt}$. **(3 marks)**

A quantity x is varying with time t in such a way that, for $t > 0$,

$$t\frac{dx}{dt} + 2t = 3x.$$

(b) Using the substitution in (a), show that this equation can be transformed into the equation

$$t\frac{dv}{dt} = 2(v - 1).$$ **(3 marks)**

(c) Deduce that $v - 1 = kt^2$, where k is a constant. **(5 marks)**

(d) Given that $x = 3$ when $t = 1$, find the value of x when $t = 3$. **(3 marks)**

8. (a) Sketch on the same diagram, for $0 \leq \theta \leq \pi$, the curves with polar equations

$$r = a\theta \quad \text{and} \quad r = a(1 + \cos\theta).$$ **(4 marks)**

(b) Show that, at the point P where the curves intersect, $1 < \theta < 1{\cdot}5$. **(4 marks)**

(c) Taking $1{\cdot}2$ as a first approximation, use the Newton-Raphson method once to find a better estimate of the value of θ at P, correct to 4 significant figures. **(3 marks)**

(d) Using the value of θ found in (c), estimate the area of the finite region contained between the line OP (where O is the pole), the line $\theta = \dfrac{\pi}{2}$ and the curve $r = a\theta$. **(4 marks)**