PURE MATHS 4 (A) TEST PAPER 4 : ANSWERS AND MARK SCHEME

1. Points plotted at (2, 4), (1, –2) and (10, 0) — B1 B1 M1 A1 — 4

2. Sum $= \sum (4r^2 - 4r + 1) = \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$ — B1 M1 A1

   $= \frac{4}{3}n^3 + 2n^2 + \frac{2}{3}n - 2n^2 - 2n + n = \frac{1}{3}n(4n^2 - 1)$ $\quad k = \frac{1}{3}$ — M1 A1 A1 — 6

3. (i) $x > -1$ and $2x - 1 < x + 1$ or $x < -1$ and $2x - 1 > x + 1$ — M1 M1 A1

   Solution set is $-1 < x < 2$ — M1 A1

   (ii) Complement of set in (i), but undefined at –1, so $x < -1$ or $x \geq 2$ — M1 A1 — 7

4. (a) f(4) = 0·147, f(5) = –0·075, so root between 4 and 5 — M1 A1

   (b) Start with 4·5, then 4·75, then 4·625; α = 4·63 to 3 s.f. — B1 B1 M1 A1

   (c) 0·00279 — M1 A1 — 8

5. $y'' - 6y' + 9y = 0$ Aux eqn. $u^2 - 6u + 9 = 0$ has root $u = 3$ — B1 M1 A1

   $y = (a + bx)\,e^{3x}$ $y(0) = 3 : a = 3$ — M1 A1

   $y' = [3(a + bx) + b]e^{3x}$ $y'(0) = 2 : 9 + b = 2$ $\quad b = -7$ — B1 M1 A1

   $y = (3 - 7x)e^{3x}$ — A1 — 9

6. (a) (i) $wz = (a - b) + (ab + 1)\mathrm{i}$ (ii) $(wz)^* = (a - b) - (ab + 1)\mathrm{i}$ — M1 A1 A1

   (iii) $\frac{z}{w} = \frac{(1 + bi)(a - \mathrm{i})}{(a + \mathrm{i})(a - \mathrm{i})} = \frac{a + b}{a^2 + 1} + \frac{ab - 1}{a^2 + 1}\mathrm{i}$ — M1 A1 A1

   (b) $a^2 + 1 = 10$ $\quad a^2 = 9$ $\quad a > 0$, so $a = 3$ — B1 M1 A1

   $|z| = \sqrt{50}$ $\quad b^2 + 1 = 50$ $\quad b^2 = 4$ $\quad b > 0$, so $b = 7$ — B1 M1 A1 — 12

7. (a) $\frac{dx}{dt} = \frac{d}{dt}(vt) = t\frac{dv}{dt} + v\frac{dt}{dt} = v + t\frac{dv}{dt}$ — M1 A1 A1

   (b) $t\left(v + t\frac{dv}{dt}\right) + 2t = 3vt$ $\quad v + t\frac{dv}{dt} + 2 = 3v$ $\quad t\frac{dv}{dt} = 2v - 2$ — M1 A1 A1

   (c) Separating variables : $\frac{1}{v-1}\,dv = \frac{2}{t}\,dt$ $\quad \ln(v - 1) = 2\ln t + c$ — M1 A1 M1

   $\ln(v - 1) = \ln(kt^2)$ $\quad v - 1 = kt^2$ — A1 A1

   (d) $x = t(kt^2 + 1)$ $\quad x(1) = 3 : k = 2$ $\quad x(3) = 57$ — M1 A1 A1 — 14

8. (a) Curves sketched : spiral and cardioid — B2 B2

   (b) Where curves meet, $\theta = 1 + \cos\theta$ Let $f(\theta) = 1 + \cos\theta - \theta$ — M1 M1

   f(1) = cos 1 > 0, $\quad$ f(1·5) = cos 1·5 – 0·5 < 0, so root in (1, 1·5) — M1 A1

   (c) $f'(\theta) = -\sin\theta - 1$ $\quad \theta \approx 1{\cdot}2 - f(1{\cdot}2)/f'(1{\cdot}2) = 1{\cdot}284$ — B1 M1 A1

   (d) Area $\approx \frac{a^2}{2}\left(\int_{1{\cdot}284}^{\pi/2} \theta^2\,d\theta\right) = \frac{a^2}{6}\left[\theta^3\right]_{1{\cdot}284}^{\pi/2} \approx 0{\cdot}29a^2$ — M1 M1 A1 A1 — 15