PURE MATHS 4 (A) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1. Graphs intersect only where $1/x = 2x + 1$, $x > 0$ — B1 B1
   $2x^2 + x - 1 = 0$ Positive root is $x = 1/2$ Solution set $x \geq 1/2$ — M1 A1 — 4

2. Sum is $\sum (r^3 + 6r^2 + 12r + 8)$ — B1
   $= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + 6n(n+1) + 8n$ — M1 A1
   Putting $n = 25$ gives $105625 + 33150 + 3900 + 200 = 142875$ — M1 A1 — 5

3. (a) Modulus = 3, argument = $-\pi/3$ — B1 M1 A1
   (b) Roots are $3(1/2 \pm (i\sqrt{3})/2)$. Sum = 3, product = $9(1/4 + 3/4) = 9$ — B1 M1 M1
   Hence $b = -3$, $c = 9$ — A1 (both) — 7

4. Equation is $\frac{dP}{dT} + \frac{1}{2}P = 2T$ Int. factor = $e^{T/2}$ — B1 M1 A1
   $e^{\frac{T}{2}}\frac{dP}{dT} + \frac{1}{2}e^{\frac{T}{2}}P = 2Te^{\frac{T}{2}}$ $\frac{d}{dT}\left(Pe^{\frac{T}{2}}\right) = 2Te^{\frac{T}{2}}$ — M1 A1
   $Pe^{\frac{T}{2}} = (4T - 8)e^{\frac{T}{2}} + c$ $P = 4T - 8 + ce^{-\frac{T}{2}}$ — M1 A1 A1 — 8

5. (a) $f(0) = -1$, $f(1) = 0{\cdot}29$; $f(9) = 0{\cdot}28$, $f(10) = -0{\cdot}38$ — B1 B1
   (b) Root $\approx 9 + 0{\cdot}282/0{\cdot}643 = 9{\cdot}43$ — M1 A1 A1
   (c) $f'(x) = \frac{2}{3}\cos\frac{x}{3} + e^{-x}$ $0{\cdot}5 - f(0{\cdot}5)/f'(0{\cdot}5) = 0{\cdot}7174$ — B1 M1 A1
   $0{\cdot}7174 - f(0{\cdot}7174)/f'(0{\cdot}7174) = 0{\cdot}72999 = 0{\cdot}73$ to 2 d.p. — M1 A1 — 10

6. (a) (i) $w - z = 19 - 5i$ (ii) $wz = -84 - 13i$ (iii) $\frac{w}{z} = -\frac{36}{25} + \frac{77}{25}i$ — B1 M1 A1 M1 A1
   (b) Points plotted Line from O to $w - z$ is — B2
   equal and parallel to line joining points rep. z and w — B1
   (c) $r(19 - 5i) + s(-84 - 13i)$ real : $-5r - 13s = 0$ e.g. $r = -13$, $s = 5$ — M1 A1 A1 — 11

7. Aux. eqn. is $u^2 + 9u - 22 = 0$, with roots $u = 2$, $u = -11$ — M1 A1
   C.F. is $y = ae^{2x} + be^{-11x}$ Take P.I. $= px^2 + qx + r$ — A1 M1
   $y'' + 9y' - 22y = 2p + 9(2px + q) - 22(px^2 + qx + r)$ — M1
   $= -22px^2 + (18p - 22q)x + (2p + 9q - 22r)$ $121 = -22p : p = -11/2$ — M1 A1
   $-99 - 22q = 11 : q = -5$ $-11 - 45 - 22r = 0$ $r = -28/11$ — A1
   $y = ae^{2x} + be^{-11x} - 11x^2/2 - 5x - 28/11$ $y(0) = 1 : a + b = 39/11$ — A1 M1
   $y' = 2ae^{2x} - 11be^{-11x} - 11x - 5$ $y'(0) = 8 : 2a - 11b = 13$ — A1 M1
   $a = 4$, $b = -5/11$ $y = 4e^{2x} - 5e^{-11x}/11 - 11x^2/2 - 5x - 28/11$ — A1 A1 — 14

8. (a) Curve sketched : loop from Pole to Pole — B2
   (b) $y = r\sin\theta = 2a\sin^2\theta\cos\theta$ $dy/d\theta = -2a\sin^3\theta + 4a\sin\theta\cos^2\theta$ — B1 M1 A1
   $= 0$ at required point $-3\sin^3\theta + 2\sin\theta = 0$ — M1 A1
   $\sin\theta = 0$ or $\sin\theta = \sqrt{(2/3)}$ $\theta = 0{\cdot}955$ $(0{\cdot}942a, 0{\cdot}955)$ — M1 A1
   (c) Area $= \frac{1}{2}\int_0^{\pi/2} a^2\sin^2 2\theta\, d\theta = \frac{a^2}{4}\int_0^{\pi/2} \cos 4\theta - 1\, d\theta$ — M1 A1 M1 A1
   $= \frac{a^2}{16}[4\sin 4\theta - 4]_0^{\pi/2} = -\frac{\pi a^2}{8}$ Area $= \frac{\pi a^2}{8}$ — A1 M1 A1 — 16