PURE MATHS 4 (A) TEST PAPER 1 : ANSWERS AND MARK SCHEME

1. (a) Curve sketched : three 'leaves' (b) $r_{max} = a$ B4 B1 5

2. $(x^2 + 2)^2 < (x - 4)^2$ $x^4 + 4x^2 + 4 < x^2 - 8x + 16$ M1 A1
$x^4 + 3x^2 + 8x - 12 < 0$ $(x - 1)(x + 2)(x^2 - x + 6) < 0$ M1 A1
Last factor is always > 0, so solution set is $-2 < x < 1$ M1 A1 6

3. (a) f(0) = –1, f(1) = 1·44, so root between 0 and 1 M1 A1
(b) $f'(x) = 2e^x - 2x$ $0{\cdot}5 - (2e^{1/2} - 3{\cdot}25)/(2e^{1/2} - 1) = 0{\cdot}479$ B1 M1 A1 A1 6

4. (a) arg $w = \pi/4$ arg $z = -\pi/3$ B1 M1 A1
(b) $r = |wz| = |w| \times |z| = 2\sqrt{2}$ $\theta = \arg(wz) = \pi/4 - \pi/3 = -\pi/12$ M1 A1 M1 A1
(c) Modulus = $(\sqrt{2})/4$, argument = $\pi/12$ B1 B1 9

5. (a) $\Sigma(9r^2 + 3r - 2) = 9 \times \frac{1}{6}n(n + 1)(2n + 1) + 3 \times \frac{1}{2}n(n + 1) - 2n$ M1 A1 A1
$= 3n^3 + 6n^2 + n = n(3n^2 + 6n + 1)$ M1 A1
(b) With S_n as in (a), we need $S_{50} - S_9 + 100 - 18$ M1 A1
$= 390\,050 - 2682 + 82 = 387\,450$ M1 A1 9

6. (a) $\frac{dy}{dx} - \frac{1}{2x}y = \frac{1}{2}$ Integrating factor = $e^{(-\ln x)/2} = \frac{1}{\sqrt{x}}$ B1 B1
$x^{-\frac{1}{2}}\frac{dy}{dx} - \frac{1}{2}x^{-\frac{3}{2}}y = \frac{1}{2}x^{-\frac{1}{2}}$ $\frac{d}{dx}\left(x^{-1/2}y\right) = \frac{1}{2}x^{-1/2}$ M1 A1 A1
$x^{-1/2}y = x^{1/2} + c$ $y = x + c\sqrt{x}$ M1 A1
$y(1) = -2 : c = -3$ $y = x - 3\sqrt{x}$ A1
(b) $x = 4 : y = 4 - 6 = -2$ M1 A1 10

7. (a) (i) $A = (5, 0)$ $B = (\pi, \pi/2)$ B1; B1 B1
(ii) At C, $r\cos\pi/4 = 5/2$ $r = (5\sqrt{2})/2$ $C = ((5\sqrt{2})/2, \pi/4)$ M1 A1
(b) $x = r\cos\theta = 2\theta\cos\theta$ $dx/d\theta = 2\cos\theta - 2\theta\sin\theta$ M1 A1
$= 0$ at D $\theta\sin\theta = \cos\theta$ $\tan\theta = 1/\theta$ M1 A1
(c) Area = $\frac{1}{2}\int_0^{\pi/2} r^2\,d\theta = \frac{1}{2}\int_0^{\pi/2} 4\theta^2\,d\theta = \left[\frac{4\theta^3}{3}\right]_0^{\pi/2} = \frac{\pi^3}{12}$ M1 A1 M1 A1 A1
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8. (a) Aux. eqn. is $2u^2 - 3u - 2 = 0$, with roots $u = 2$, $u = -1/2$ M1 A1
so C.F. = $ae^{2x} + be^{-x/2}$ Let P.I. be $y = p\cos x + q\sin x$ A1 M1
Then $2(-p\cos x - q\sin x) - 3(-p\sin x + q\cos x)$
$- 2(p\cos x + q\sin x) = 25\cos x$ M1
$-4p - 3q = 25$, $3p - 4q = 0$ $p = -4$, $q = -3$ A1 A1
General solution is $y = ae^{2x} + be^{-x/2} - 4\cos x - 3\sin x$ A1
(b) $y' = 2ae^{2x} - (1/2)be^{-x/2} + 4\sin x - 3\cos x$ M1 A1
$a + b - 4 = 1$, $2a - b/2 - 3 = -1$ $a = 9/5$, $b = 16/5$ M1 A1
$y = 9e^{2x}/5 + 16e^{-x/2}/5 - 4\cos x - 3\sin x$ A1
(c) When $x = \pi$, $y = (9e^{2\pi} + 16e^{-\pi/2})/5 + 4 = 970$ (to 2 s.f.) M1 A1 A1 16