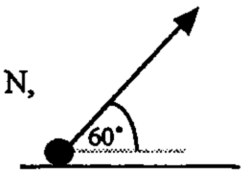
Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A particle of mass 0·6 kg moves in a horizontal circle with constant angular speed 1·5 radians per second. If the force directed towards the centre of the circle has magnitude 0·27 N, find the radius of the circular path. **(3 marks)**

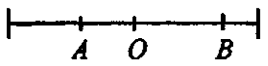
2. The diagram shows a particle of mass 0·7 kg resting on a rough horizontal table. The coefficient of friction between the particle and the table is 0·25. A light elastic string, of natural length 50 cm and modulus of elasticity 6·86 N, is attached to the particle. The string is kept at an angle of 60° to the horizontal and is gradually extended by pulling on it until the particle moves.



Show that the particle starts to move when the extension in the string is 17 cm. **(8 marks)**

3. A smooth circular hoop of radius 1 m, with centre O, is fixed in a vertical plane. A small ring Q, of mass 0·1 kg, is threaded onto the hoop and held so that the angle $QOH = 30°$, where H is the highest point of the hoop. Q is released from rest at this position. Find, in terms of g,
   (a) the horizontal and vertical components of the acceleration of Q when it reaches the lowest point of the hoop; **(5 marks)**
   (b) the magnitude of the reaction between Q and the hoop at this lowest point. **(3 marks)**

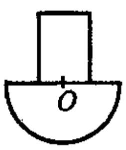
4. A particle P moves with simple harmonic motion in a straight line, with the centre of motion at the point O on the line. A and B are on opposite sides of O, with $OA = 4$ m, $OB = 6$ m.



When passing through A and B, P has speed 6 ms^{-1} and 4 ms^{-1} respectively.
   (a) Find the amplitude of the motion. **(6 marks)**
   (b) Show that the period of motion is 2π s. **(3 marks)**

5. (a) Prove that the centre of mass of a uniform solid hemisphere of radius r is at a distance $\frac{3r}{8}$ from its plane face. **(7 marks)**

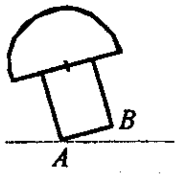
A solid cylinder of radius $\frac{3r}{4}$ and height kr, where $k < 1$, is welded to a uniform hemisphere of radius r made of the same material, so that their axes of symmetry coincide. The figure shows the cross section of the resulting solid. If the centre of mass of this solid is at O, the centre of the plane face of the hemisphere,



   (b) find the value of k. **(4 marks)**

[Turn over ...

5. continued...



The solid now stands on the base of the cylindrical portion, whose diameter is AB, and is gently tilted about A.

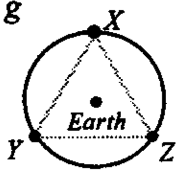
(c) Find the angle between AB and the horizontal when it is on the point of toppling. **(3 marks)**

6. The gravitational attraction F N between two point masses m_1 kg and m_2 kg at a distance x m apart is given by $F = \frac{km_1m_2}{x^2}$, where k is a constant. Given that a small body of mass 1 kg experiences a force of g N at the surface of the Earth, which has radius R m and mass M kg,

(a) show that $k = \frac{gR^2}{M}$. **(2 marks)**

A small communications satellite of mass m kg is put into a circular orbit of radius r m around the Earth. Modelling the Earth as a particle of mass M kg, and using the value of k from (a),

(b) prove that the period of rotation, T s, of the satellite is given by $T = \frac{2\pi}{R}\sqrt{\frac{r^3}{g}}$. **(4 marks)**



To cover transmission to any point on the Earth, three small satellites X, Y and Z, each of mass m kg, are placed in a common circular orbit of radius r and form an equilateral triangle as shown.

(c) Show on a copy of the diagram the direction of the three forces acting on X. **(1 mark)**

(d) State, with a reason, the direction of the resultant force on X. **(2 marks)**

(e) Show that the period of rotation of X is given by $T\sqrt{\frac{3M}{3M + m\sqrt{3}}}$ s, where T s is the period found in (b). **(7 marks)**

7. One end of a light elastic string, of natural length $3l$ m, is attached to a fixed point O. A particle of mass m kg is attached to the other end of the string. When the particle hangs freely in equilibrium, the string is extended by a length of l m. The particle is then pulled down through a further distance $2l$ m and released from rest.

(a) Prove that as long as the string is taut, the particle performs simple harmonic motion about its equilibrium position. **(5 marks)**

(b) Show that the time between the release of the particle and the instant when the string becomes slack is $\frac{2}{3}\pi\sqrt{\frac{l}{g}}$ s. **(4 marks)**

(c) Find the greatest height reached by the particle above its point of release. **(4 marks)**

(d) Show that the time T s taken to reach this greatest height from the moment of release is given by $T = \left(\frac{2\pi}{3} + \sqrt{3}\right)\sqrt{\frac{l}{g}}$. **(4 marks)**