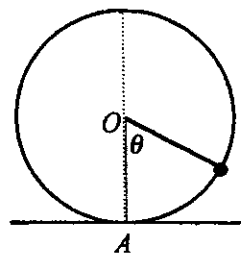


Take $g = 9.8 \text{ ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. A particle of mass m kg is attached to one end of a light inextensible string of length l m whose other end is fixed to a point O . The particle is made to move in a vertical circle with centre O , with constant angular velocity $\omega \text{ rad s}^{-1}$. At a certain instant it is in the position shown, where the string makes an angle θ radians with the downward vertical through O .

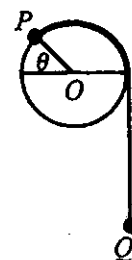


- (a) Find an expression, in terms of m , l and ω , for the kinetic energy of the particle at this instant. (2 marks)
- (b) Find an expression, in terms of m , g , l and θ , for the potential energy of the particle relative to the horizontal plane through the lowest point A . (2 marks)
- (c) Determine the position of the particle when the rate of increase of its total energy, with respect to time, is a maximum. (3 marks)
2. A particle moves along a straight line in such a way that its displacement x m from a fixed point O on the line, at time t seconds after it leaves O , is given by $x = p \sin \omega t + q \cos \omega t$ where p , q and ω are constants.
- (a) Show that the motion of the particle is simple harmonic. (5 marks)
- (b) If the particle leaves O with speed 15 ms^{-1} , and $\omega = 3$, find the amplitude of the motion. (2 marks)
3. A particle P of mass 0.2 kg moves in a horizontal circle on one end of an elastic string whose other end is attached to a fixed point O . The angular velocity of P is $\pi \text{ rad s}^{-1}$. The natural length of the string is 1 m and, while P is in motion, the distance $OP = 1.15$ m.
- (a) Calculate, to 3 significant figures, the modulus of elasticity of the string. (6 marks)
- The motion now ceases and P hangs at rest vertically below O .
- (b) Show that the extension in the string in this position is about 13 cm. (3 marks)
4. A small stone P of mass m kg is attached to one end of a light elastic string of modulus $3mg$ N and natural length $2l$ m. The other end of the string is fixed to a point O at a height $3l$ m above a horizontal surface. P is released from rest at O ; it hits the surface and rebounds to a height of $2l$ m. The coefficient of restitution between P and the surface is e .
- Calculate the value of e . (9 marks)
- State one assumption (other than the string being light) that you have used in your solution. (1 mark)

5. A small sphere S , of mass m kg is released from rest at the surface of a liquid in a right circular cylinder whose axis is vertical. When S is moving downwards with speed v ms^{-1} , the viscous resistive force acting upwards on it has magnitude v^2 N.

- (a) Write down a differential equation for the motion of S , clearly defining any symbol(s) that you introduce. (4 marks)
- (b) Find, in terms of m , the distance S has fallen when its speed is $\sqrt{\frac{mg}{2}}$ ms^{-1} . (9 marks)

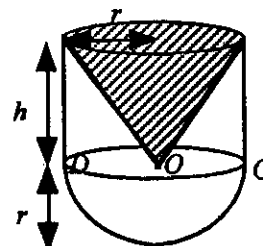
6. The diagram shows two identical particles, each of mass m kg, connected by a thin, light inextensible string. P slides on the surface of a smooth right circular cylinder fixed with its axis, through O , horizontal. Q moves vertically. OP makes an angle θ radians with the horizontal.



The system is released from rest in the position where $\theta = 0$.

- (a) Show that the vertical distance moved by Q is $\frac{\theta}{\sin \theta}$ times the vertical distance moved by P . (4 marks)
- (b) In the position where $\theta = \frac{\pi}{6}$, prove that the reaction of the cylinder on P has magnitude $(1 - \frac{\pi}{6})mg$ N. (9 marks)

7. A container consists of two sections made from the same material : a hollow portion formed by removing a cone (shaded in the figure) from a solid cylinder of radius r and height h , and a solid hemisphere of radius r . The vertex of the removed cone coincides with the centre O of the horizontal plane face of the hemisphere. CD is a diameter of this plane face.



- (a) Show that the distance of the centre of mass of the container from the plane face of the hemisphere is $\left| \frac{3}{8}(h-r) \right|$. Explain why the modulus sign is necessary. (9 marks)
- (b) Find the ratio $h : r$ in each of the following cases :
- (i) When the container is suspended from the point C , the angle made by CD with the vertical is equal to the angle which CD would make with the vertical if the hemisphere *alone* were suspended from C . (4 marks)
- (ii) The container is able to stand without toppling in any position when it is placed with the surface of the hemispherical part in contact with a smooth horizontal table. (3 marks)