MECHANICS 3 (A) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1.	(a) $T\cos 60° = mg$, so $T = 2mg$	$T\sin 60° = m(L\sin 60°)\omega^2$		M1 A1 M1 A1	
	Thus $\frac{mg}{l}(L-l) = 2mg$	$L - l = 2l$	$L = 3l$	M1 A1	
	(b) $2mg = m(3l)\omega^2$	$\omega^2 = \frac{2g}{3l}$	$\omega = \sqrt{\frac{2g}{3l}}$	M1 A1	8
2.	(a) $mv\frac{dv}{dx} = -(mg + mf(x))$	$v\frac{dv}{dx} = -g - f(x)$		M1 A1	
	$v^2 = 2e^{-2gx} - 1$, so $2v\frac{dv}{dx} = -4ge^{-2gx}$	$-2ge^{-2gx} = -g - f(x)$		M1 A1	
	$f(x) = g(2e^{-2gx} - 1)$	(b) $v = 0$ when $2e^{-2gx} = 1$	$x = \frac{1}{2g}\ln 2$	A1; M1 A1	
	(c) W.D. $= m[-e^{-2gx} - gx]_0^{(\ln 2)/2g} = m(-e^{-\ln 2} - \frac{1}{2}\ln 2 + 1) = \frac{1}{2}m(1 - \ln 2)$			M1 A1 A1	10
3.	Speed $< u$, so friction F acts up.	$R\cos\theta + F\sin\theta = mg$ (1)		B1 M1 A1	
	$R\sin\theta - F\cos\theta = \frac{mu^2}{4r}$ (2)	(1) x $\sin\theta$, (2) x $\cos\theta$, subtract :		M1 A1 A1 M1	
	$F = mg\sin\theta - \frac{mu^2}{4r}\cos\theta = mg\sin\theta - \frac{mg}{4}\tan\theta\cos\theta = \frac{3}{4}mg\sin\theta$			A1 A1 A1	10
4.	E.P.E. changes from $2\frac{\lambda}{2l}l^2(\sec 30° - 1)^2$ to $2\frac{\lambda}{2l}l^2(\sec 60° - 1)^2$			M1 A1 A1	
	Gain in E.P.E. $= \lambda l[(\sec 60° - 1)^2 - (\sec 30° - 1)^2] = \lambda l\left(\frac{4}{\sqrt{3}} - \frac{4}{3}\right)$			M1 A1	
	Loss in grav. P.E. $= mgl(\tan 60° - \tan 30°) = \frac{2mgl}{\sqrt{3}}$			M1 A1	
	Hence $\lambda l\left(\frac{12-4\sqrt{3}}{3\sqrt{3}}\right) = \frac{2mgl}{\sqrt{3}}$	$\lambda = \frac{3mg}{6-2\sqrt{3}}$		M1 A1 A1	10
5.	(a) (i) $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgr(1-\cos\theta)$	$v^2 = u^2 + 2gr(1-\cos\theta)$		M1 A1 A1	
	(ii) $mg\cos\theta - X = \frac{mv^2}{r}$	$X = mg\cos\theta - \frac{mu^2}{r} - 2mg(1-\cos\theta)$		M1 A1	
	$X = mg[3\cos\theta - 2 - \frac{u^2}{gr}]$			A1	
	(b) Leaves sphere when $X = 0$, i.e. when $3\cos\theta = 2 + \frac{u^2}{gr}$, etc.			M1 A1 A1	
	(c) If $u^2 \geq gr$, P leaves the surface as soon as it is projected			B2	11
6.	(a) $mg = \frac{\lambda e}{l}$ $\lambda = \frac{mgl}{e}$	SHM : $m\ddot{x} = mg - \frac{mgl}{el}(e + x)$		M1 A1	
	$\ddot{x} = -\omega^2 x = -\frac{g}{e}x$ $\omega^2 = \frac{g}{e}$	5 osc. per second, so		M1 A1 M1	
	$\omega^2 = 4\pi^2(5^2) = 100\pi^2$	Thus $\frac{g}{e} = 100\pi^2$	$\frac{\lambda}{l} = 100\pi^2 m$	A1 M1 A1	
	(b) $mg = \frac{\lambda e}{l} = 100\pi^2 me$, so $e = \frac{g}{100\pi^2}$			M1 A1	
	(c) $T_1 = \frac{\lambda}{l}l = \lambda = 100\pi^2 ml$ N			M1 A1	12
7.	(a) $\bar{x}\int_0^h \pi y^2\,dx = \int_0^h \pi y^2 x\,dx$	$y = \frac{rx}{h}$, so $\bar{x}\int_0^h \frac{r^2x^2}{h^2}dx = \int_0^h \frac{r^2x^3}{h^2}dx$		M1 A1 B1 M1	
	$\bar{x}\frac{h^3}{3} = \frac{h^4}{4}$ $\bar{x} = \frac{3h}{4}$			M1 A1 A1	
	(b) M(vertex) : $m\frac{2h}{3} + m\frac{3kh}{4} = 2mkh$	$k = \frac{8h}{15}$		M1 A1 A1	
	(c) $\tan\alpha = r \div \frac{7h}{15}$	$r = \frac{7h}{15}\tan 45° = \frac{7h}{15}$ or $0{\cdot}47h$		M1 A1 M1 A1	
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