MECHANICS 3 (A) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1. (a) Total energy is conserved throughout, so $v^2 = gr$ M1 A1

   (b) $T = \frac{mv^2}{r}$ $\frac{1}{2}mu^2 = m(2gr - gr) + \frac{1}{2}mgr$ $u^2 = 3gr$ $T = 3mg$ M1 A1 M1 A1 A1 7

2. (a) $T = \frac{2\pi}{n}$ $\frac{\pi}{2} = \frac{2\pi}{n}$ $n = 4$ M1 A1

   $v^2 = n^2(a^2 - x^2)$ $9 = 16(a^2 - 1)$ $a = 1{\cdot}25$ m M1 A1

   (b) $a_{max} = an^2 = 1{\cdot}25 \times 16 = 20$ ms^{-2} B1

   (c) $x = a \sin nt$ $0{\cdot}25 = 1{\cdot}25 \sin 4t$ $\sin 4t = 0{\cdot}2$ M1 A1

   Least t when $4t = \arcsin(0{\cdot}2)$ $t = 0{\cdot}05$ s M1 A1 9

3. (a) Grav. P.E. loss = E.P.E. gain : $3mgl = \frac{\lambda}{16l}(10l - 8l)^2$ $\lambda = 12\,mg$ M1 A1 M1 A1

   (b) P.E. loss = (K.E. + E.P.E.) gain : $2mgl = \frac{1}{2}mv^2 + \frac{12mg}{16l}(4\sqrt{5}\,l - 8l)^2$ M1 A1

   $4gl = v^2 + 24gl(1 - 4\sqrt{5})$ $v^2 = 4(24\sqrt{5} - 53)gl$ M1 A1 A1 9

4. (a) $F = ma$: $0{\cdot}8v\frac{dv}{dx} = -\frac{0{\cdot}8xv^2}{1+x^2}$ $\frac{dv}{dx} = -\frac{xv}{1+x^2}$ M1 A1

   (b) $\int \frac{dv}{v} = -\int \frac{x\,dx}{1+x^2}$ $\ln v = -\frac{1}{2}\ln(1 + x^2) + c$ $c = \ln 2$ M1 A1 M1 A1 A1

   When $x = 1$, $\ln v = \ln 2 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln 2$ $v = \sqrt{2}$ M1 A1 9

5. (a) Centre of mass of cone is $\frac{1}{4}h$ from base along axis B1

   Let $POQ = 2\alpha$. M(O) : $T(2r\cos\alpha) = \frac{3}{4}h\sin\alpha$ B1 M1 A1 A1

   $T = \frac{3h}{8r}\tan\alpha$ But $\tan\alpha = \frac{r}{h}$ so $T = \frac{3}{8}mg$ B1 M1 A1

   (b) Vertically : $S + T = mg$ $S = \frac{5}{8}mg$ M1 A1 10

6. (a) For Q : $T = mg$ For P : $T = m(0{\cdot}2)\omega^2$ B1 M1 A1

   $\omega^2 = g/0{\cdot}2 = 49$ $\omega = 7$ No. of r.p.m. $= \frac{7}{2\pi} \times 60 = 66{\cdot}8$ A1 A1

   (b) For Q : $T\sin 45° = m(0{\cdot}2)\omega_1^2$, $T\cos 45° = mg$ M1 A1 A1

   $\tan 45° = 0{\cdot}2\omega_1^2 / g$ $\omega_1^2 = 49\tan 45° = 49$ M1 A1

   For P : $T = m(0{\cdot}2)\omega^2$ But $T = mg\sqrt{2}$ so $\omega^2 = 49\sqrt{2}$ M1 A1

   $\omega^2 : \omega_1^2 = \sqrt{2} : 1$ $\omega : \omega_1 = 2^{1/4} : 1$ M1 A1 14

7. (a) At distance x from O, $T = -m\ddot{x}$ $T = \frac{kmg}{l}$ B1 B1

   Hence $\ddot{x} = -\frac{kg}{l}x$, so motion is S.H.M. M1 A1

   (b) Loss of E.P.E. = gain in K.E. : $\frac{kmg}{2l}\left(\frac{9}{16}l^2\right) = \frac{1}{2}mgl$ $k = \frac{16}{9}$ M1 A1 A1

   (c) K.E. dissipated = work done against friction : $\frac{1}{2}mgl = \mu mgx$ M1 A1

   $x = \frac{l}{2\mu}$ which must be $< l$, so $\mu > \frac{1}{2}$ M1 A1

   (d) Time from O to stop is t where $0 = \sqrt{gl} - \frac{3}{4}gt$ $t = \frac{4}{3}\sqrt{\frac{l}{g}}$ M1 A1

   $\frac{1}{4}$ period $= \frac{1}{4}\left(\frac{2\pi}{\sqrt{(16g/9l)}}\right) = \frac{3\pi}{8}\sqrt{\frac{l}{g}}$ Total time $= \left(\frac{3\pi}{8} + \frac{4}{3}\right)\sqrt{\frac{l}{g}}$ M1 M1 A1 A1 17