GCE Examinations

Advanced Subsidiary / Advanced Level

# Mechanics
# Module M3

Paper A

# MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

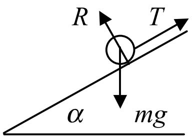
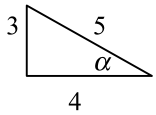
Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

*Written by Shaun Armstrong & Chris Huffer*

# M3 Paper A – Marking Guide

| | | | | |
|---|---|---|---|---|
| **1.** | *(a)* | $T = \frac{\lambda x}{l} = \frac{30 \times 0.2}{1} = 6$ N | M1 A1 | |
| | *(b)* |  resolve ↗: $T - mg\sin\alpha = ma$ | M1 A1 | |
| | |  $\therefore \sin\alpha = \frac{3}{5}$ | M1 | |
| | | $\therefore\ 6 - 0.6 \times 9.8 \times \frac{3}{5} = 0.6a$ | M1 | |
| | | giving $a = 4.12$ ms$^{-2}$ | A1 | **(7)** |
| **2.** | *(a)* | $F = ma = 0.5\, v\frac{\mathrm{d}v}{\mathrm{d}x} = 3\, x^{\frac{1}{2}}$ | M1 | |
| | | $\therefore \int v\ \mathrm{d}v = \int 6\, x^{\frac{1}{2}}\ \mathrm{d}x$ | M1 | |
| | | giving $\frac{1}{2}v^2 = 4\, x^{\frac{3}{2}} + c$ | A1 | |
| | | $x = 1, v = 2\ \ \therefore c = {}^{-}2$ | M1 | |
| | | $\therefore v^2 = 8\, x^{\frac{3}{2}} - 4$ | A1 | |
| | *(b)* | $x = 4$ gives $v^2 = 64 - 4 = 60\ \ \therefore v = \sqrt{60} = 7.7$ ms$^{-1}$ (1dp) | M1 A1 | **(7)** |
| **3.** | *(a)* | amplitude = $\frac{1}{2} \times 8 = 4$ m | B1 | |
| | | period = $\frac{2\pi}{\omega} = 12\ \ \therefore \omega = \frac{\pi}{6}$ | B1 | |
| | | $v_{max} = a\omega = 4 \times \frac{\pi}{6} = \frac{2\pi}{3}$ ms$^{-1}$ | M1 A1 | |
| | *(b)* | $x = a\sin\omega t$ | M1 | |
| | | at $P$, ${}^{-}1 = 4\sin\omega t\ \ \therefore \frac{\pi}{6}t = {}^{-}0.2527,\ t = {}^{-}0.4826$ | M1 A1 | |
| | | at $Q$, $2 = 4\sin\omega t\ \ \therefore \frac{\pi}{6}t = \frac{\pi}{6},\ t = 1$ | M1 A1 | |
| | | $\therefore$ time between = 1.48 s (3sf) | A1 | **(10)** |
| **4.** | *(a)* | $v^2 = kg - kg\,e^{-\frac{2x}{k}}\ \ \therefore 2v\frac{\mathrm{d}v}{\mathrm{d}x} = 2g\,e^{-\frac{2x}{k}}$ | M1 A2 | |
| | | $f =$ accel. $= v\frac{\mathrm{d}v}{\mathrm{d}x} = g\,e^{-\frac{2x}{k}}$ | A1 | |
| | *(b)* | when $x$ is large, $e^{-\frac{2x}{k}} \to 0$ | M1 | |
| | | $\therefore 49^2 = kg$ giving $k = \frac{49^2}{9.8} = 245$ | M1 A1 | |
| | *(c)* | $v^2 = kg - kg\,e^{-\frac{2x}{k}} = kg - kf$ | M1 A1 | |
| | | $\therefore f = g - \frac{1}{k}v^2 = 9.8 - \frac{1}{245}v^2$ | M1 A1 | **(11)** |

**5.** *(a)*

| portion | mass | $y$ | $my$ |
|---|---|---|---|
| cone | $\rho\times\frac{1}{3}\pi r^2(2r) = \frac{2}{3}\rho\pi r^3$ | $h+\frac{1}{4}(2r)=h+\frac{1}{2}r$ | $\frac{2}{3}\rho\pi r^3(h+\frac{1}{2}r)$ |
| cylinder | $\rho\pi r^2 h$ | $\frac{1}{2}h$ | $\frac{1}{2}\rho\pi r^2 h^2$ |
| firework | $\rho\pi r^2(h+\frac{2}{3}r)$ | $\overline{y}$ | $\rho\pi r^2(\frac{1}{2}h^2+\frac{2}{3}rh+\frac{1}{3}r^2)$ |

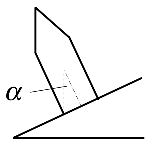
$\rho$ = mass per unit volume $\quad$ $y$ coords. taken vert. from base — M2 A4

$\rho\pi r^2(h+\frac{2}{3}r)\times\overline{y} = \rho\pi r^2(\frac{1}{2}h^2+\frac{2}{3}rh+\frac{1}{3}r^2)$ — M1

$\therefore\ 2(3h+2r)\times\overline{y} = 3h^2+4rh+2r^2$ — M1

giving $\overline{y} = \frac{3h^2+4hr+2r^2}{2(3h+2r)}$ — A1

*(b)*




$h = 4r\ \therefore\ \overline{y} = \frac{33}{14}r$ — M1

$\tan\alpha = r \div (\frac{33}{14}r) = \frac{14}{33}$ — M1 A1

$\therefore\ \alpha = 23°$ (nearest degree) — A1 **(13)**

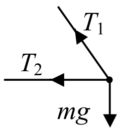
---

**6.** *(a)* string taut $\therefore PR = a,\ PR^2 + QR^2 = a^2 + 3a^2 = 4a^2 = PQ^2$ — M1

by converse of Pythag. $\angle PRQ = 90°$ — A1

*(b)* $\sin\angle PQR = \frac{a}{2a} = \frac{1}{2}\ \therefore\ \angle PQR = 30°$ — B1

*(c)* (i)




resolve ↑: $T_1\sin 60 - mg = 0$ — M1 A1

$\therefore\ T_1 = \frac{2mg}{\sqrt{3}}$ (or $\frac{2}{3}\sqrt{3}\,mg$) — A1

(ii) resolve ←: $T_2 + T_1\cos 60 = \frac{mv^2}{r}$ — M1 A1

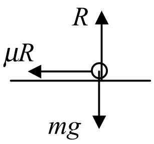
$\therefore\ T_2 = \frac{mu^2}{a} - \frac{1}{2}\times\frac{2mg}{\sqrt{3}} = \frac{mu^2}{a} - \frac{mg}{\sqrt{3}}$ (or $\frac{mu^2}{a} - \frac{1}{3}\sqrt{3}\,mg$) — M1 A1

*(d)* $PR$ taut $\therefore T_2 \geq 0$ — M1

giving $\frac{mu^2}{a} \geq \frac{mg}{\sqrt{3}}$ so $u^2 \geq \frac{ga}{\sqrt{3}}$ — M1 A1 **(13)**

---

**7.** *(a)*




resolve ↑: $R - mg = 0\ \therefore R = 2g$ — M1 A1

friction $= \mu R = \frac{10}{49}\times 2\times 9.8 = 4$ — A1

work-energy:

work done = loss of KE – gain of EPE — M1

$\therefore\ Fs = \frac{1}{2}mu^2 - \frac{\lambda x^2}{2l}$ — M1

so $4d = \frac{1}{2}\times 2\times 5^2 - \frac{50(d-1)^2}{2\times 1}$ — A1

$\therefore\ 4d = 25 - 25(d^2 - 2d + 1)$ — M1

giving $25d^2 - 46d = 0,\ d(25d - 46) = 0$ — M1

$\therefore\ d = 0$ (initially) or $\frac{46}{25} = 1.84$ m — A1

*(b)* work-energy: work done = loss of EPE – gain of KE — M1

$\therefore\ 4\times\frac{46}{25} = \frac{50\times(\frac{21}{25})^2}{2\times 1} - \frac{1}{2}\times 2\times v^2$ — M1 A1

giving $21^2 = (4\times 46) + 25v^2$ — M1

so $v^2 = \frac{257}{25}\ \therefore\ v = 3.2$ ms$^{-1}$ (2sf) — A1 **(14)**

---

Total **(75)**

# Performance Record – M3 Paper A

| Question no. | **1** | **2** | **3** | **4** | **5** | **6** | **7** | Total |
|---|---|---|---|---|---|---|---|---|
| Topic(s) | elastic spring | variable force | SHM | variable accel. | centre of mass, equilm. | circular motion | elastic string, EPE, work done | |
| Marks | 7 | 7 | 10 | 11 | 13 | 13 | 14 | 75 |
| Student | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |