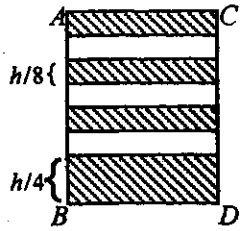
Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A small bead is threaded onto a smooth circular hoop, of radius r m, fixed in a vertical plane. It is projected with speed u ms^{-1} from the lowest point of the hoop. Find u in terms of g and r if

   (a) the bead just reaches the highest point of the hoop, **(3 marks)**

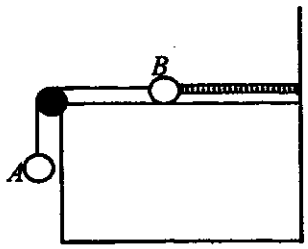
   (b) the reaction on the bead is zero when it is at the highest point of the hoop. **(4 marks)**

2. An ornamental tower is made from a solid right circular cylinder of mass M and height h by removing three identical cylindrical sections, each of height $\frac{h}{8}$, equally spaced above a base of height $\frac{h}{4}$, as shown. The tower is held in position by light, thin vertical strips AB and CD.



   Find the distance of the centre of mass of the tower from its horizontal base. **(7 marks)**

3. Two particles A and B, of masses M kg and m kg respectively, are connected by a light inextensible string passing over a smooth fixed pulley. B is placed on a smooth horizontal table and A hangs freely, as shown. B is attached to a spring of natural length l m and modulus of elasticity λ N, whose other end is fixed to a vertical wall.



   The system starts to move from rest when the string is taut and the spring neither extended nor compressed. A does not reach the ground, nor does B reach the pulley, during the motion.

   (a) Show that the maximum extension of the spring is $\frac{2Mgl}{\lambda}$ m. **(3 marks)**

   (b) If $M = 3$, $m = 1{\cdot}5$ and $\lambda = 35l$, find the speed of A when the extension in the spring is $0{\cdot}5$ m. **(6 marks)**

4. A particle P of mass m kg moves along a straight line under the action of a force of magnitude $\frac{km}{x^2}$ N, where k is a constant, directed towards a fixed point O on the line, where $OP = x$ m. P starts from rest at A, at a distance a m from O. When $OP = x$ m, the speed of P is v ms^{-1}.

   (a) Show that $v = \sqrt{\frac{2k(a-x)}{ax}}$. **(6 marks)**

   B is the point half-way between O and A. When $k = \frac{1}{2}$ and $a = 1$, the time taken by P to travel from A to B is T seconds

   Assuming the result that, for $0 \le x \le 1$, $\int \sqrt{\frac{x}{1-x}}\,dx = \arcsin(\sqrt{x}) - \sqrt{(x-x^2)} + \text{constant}$,

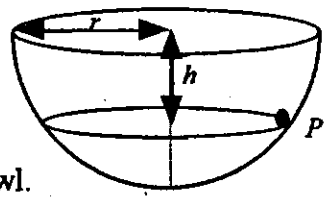
   (b) find the value of T. **(5 marks)**

5. A car moves round a circular racing track of radius 100 m, which is banked at an angle of 4° to the horizontal.

(a) Show that when its speed is $8{\cdot}28\ \text{ms}^{-1}$, there is no sideways force acting on the car. **(4 marks)**

(b) When the speed of the car is $12{\cdot}5\ \text{ms}^{-1}$, find the smallest value of the coefficient of friction between the car and the track which will prevent side-slip. **(9 marks)**

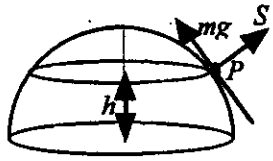
6. The diagram shows a particle P of mass m kg moving on the inner surface of a smooth fixed hemispherical bowl of radius r m which is fixed with its axis vertical. P moves at a constant speed in a horizontal circle, at a depth h m below the top of the bowl.



(a) Show that the force R exerted on P by the bowl has magnitude $\frac{mgr}{h}$ N. **(4 marks)**

(b) Find, in terms of g, h and r, the constant speed of P. **(4 marks)**

The bowl is now inverted and P moves on the smooth outer surface at a height h above the plane face under the action of a force of magnitude mg applied tangentially as shown. The reaction of the surface of the sphere on P now has magnitude S N.



(c) Given that $r = 2h$, prove that $S < \frac{1}{6}R$. **(5 marks)**

7. A particle P of mass m kg is fixed to one end of a light elastic string of modulus mg N and natural length l m. The other end of the string is attached to a fixed point O on a rough horizontal table. Initially P is at rest in limiting equilibrium on the table at the point X where $OX = \frac{5l}{4}$ m.

(a) Find the coefficient of friction between P and the table. **(2 marks)**

P is now given a small displacement x m horizontally along OX, away from O. While P is in motion, the frictional resistance remains constant at its limiting value.

(b) Show that as long as the string remains taut, P performs simple harmonic motion with X as the centre. **(4 marks)**

If P is held at the point where the extension in the string is l m and then released,

(c) show that the string becomes slack after a time $\left(\frac{\pi}{2} + \arcsin\left(\frac{1}{3}\right)\right)\sqrt{\frac{l}{g}}$ s. **(5 marks)**

(d) Determine the speed of P when it reaches O. **(4 marks)**

