Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

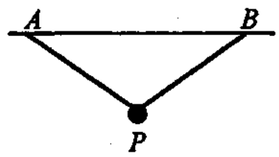
1. A particle P of mass m kg moves in a horizontal circle at one end of a light elastic string of natural length l m and modulus of elasticity mg N. The other end of the string is attached to a fixed point O. Given that the string makes an angle of 60° with the vertical,
   (a) show that $OP = 3l$ m. **(4 marks)**
   (b) Find, in terms of l and g, the angular speed of P. **(4 marks)**

2. A particle P of mass m kg moves vertically upwards under gravity, starting from ground level. It is acted on by a resistive force of magnitude m f(x) N, where f(x) is a function of the height x m of P above the ground. When P is at this height, its upward speed v ms^{-1} is given by $v^2 = 2e^{-2gx} - 1$.
   (a) Write down a differential equation for the motion of P and hence determine f(x) in terms of g and x. **(5 marks)**
   (b) Show that the greatest height reached by P above the ground is $\frac{1}{2g}\ln 2$ m. **(2 marks)**

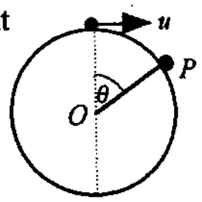
   Given that the work, in J, done by P against the resisting force as it moves from ground level to a point H m above the ground is equal to $\int_0^H m\,\text{f}(x)\,dx$,
   (c) show that the total work done by P against the resistance during its upward motion is $\frac{1}{2}m(1 - \ln 2)$ J. **(3 marks)**

3. A car of mass m kg moves round a curve of radius r m on a road which is banked at an angle θ to the horizontal. When the speed of the car is u ms^{-1}, the car experiences no sideways frictional force. Given that $\tan\theta = \dfrac{u^2}{gr}$, show that the sideways frictional force on the car when its speed is $\dfrac{u}{2}$ ms^{-1} has magnitude $\dfrac{3}{4}mg\sin\theta$ N. **(10 marks)**

4. Two light elastic strings, each of length l m and modulus of elasticity λ N,are attached to a particle P of mass m kg. The other ends of the strings are attached to fixed points A and B on the same horizontal level, where $AB = 2l$ m. P is held vertically below the mid-point of AB, with each string taut and inclined at 30° to the horizontal, and released from rest. Given that P comes to instantaneous rest when each string makes an angle of 60° with the horizontal, show that $\lambda = \dfrac{3mg}{6 - 2\sqrt{3}}$. **(10 marks)**



5. A particle P is projected horizontally with speed u ms^{-1} from the highest point of a smooth sphere of radius r m and centre O. It moves on the surface in a vertical plane, and at a particular instant the radius OP makes an angle θ with the upward vertical, as shown. At this instant P has speed v ms^{-1} and the magnitude of the reaction between P and the sphere is X N.



(a) Assuming that $u^2 < gr$, show that (i) $v^2 = u^2 + 2gr(1 - \cos\theta)$, **(2 marks)**

(ii) $X = mg\left(3\cos\theta - 2 - \frac{u^2}{gr}\right)$. **(4 marks)**

(b) Show that P leaves the surface of the sphere when $\cos\theta = \dfrac{u^2 + 2gr}{3gr}$. **(3 marks)**

(c) Discuss what happens if $u^2 \geq gr$. **(2 marks)**

6. A particle P of mass m kg hangs in equilibrium at one end of a light spring, of natural length l m and modulus of elasticity λ N, whose other end is fixed at a point vertically above P. In this position the length of the spring is $(l + e)$ m. When P is displaced vertically through a small distance and released, it performs simple harmonic motion with 5 oscillations per second.

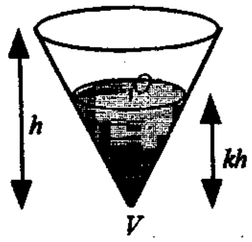
(a) Show that $\dfrac{\lambda}{l} = 100\pi^2 m$. **(8 marks)**

(b) Express e in terms of g. **(2 marks)**

(c) Determine, in terms of m and l, the magnitude of the tension in the spring when it is stretched to twice its natural length. **(2 marks)**

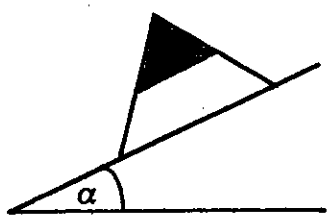
7. (a) Prove that the centre of mass of a uniform solid right circular cone of height h and base radius r is at a distance $\dfrac{3h}{4}$ from the vertex. **(7 marks)**

An item of confectionery consists of a thin wafer in the form of a hollow right circular cone of height h and mass m, filled with solid chocolate, also of mass m, to a depth of kh as shown. The centre of mass of the item is at O, the centre of the horizontal plane face of the chocolate.



(b) Show that $k = \dfrac{8h}{15}$. **(3 marks)**

In the packaging process, the cone has to move on a conveyor belt inclined at an angle α to the horizontal as shown. If the belt is rough enough to prevent sliding, and the maximum value of α for which the cone does not topple is 45°,



(c) find the radius of the base of the cone in terms of h. **(4 marks)**

