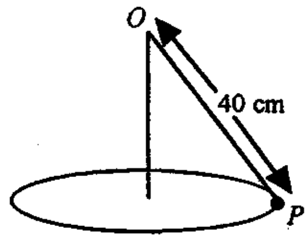
Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

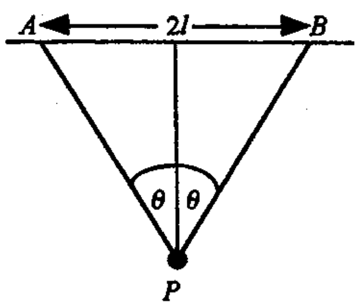
1. A particle of mass m kg moves in a horizontal straight line. Its initial speed is u ms^{-1} and the only force acting on it is a variable resistance of magnitude mkv N, where v ms^{-1} is the speed of the particle after t seconds and k is a constant.
Show that $v = ue^{-kt}$. **(7 marks)**

2. A particle P of mass m kg moves in a horizontal circle at one end of a light inextensible string of length 40 cm, as shown.
The other end of the string is attached to a fixed point O.
The angular velocity of P is ω rad s^{-1}.
If the angle θ which the string makes with the vertical must not exceed 60°, calculate the greatest possible value of ω. **(7 marks)**



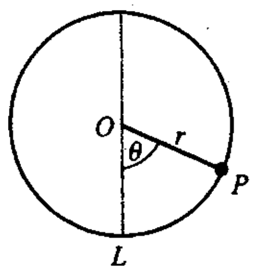
3. A particle P of mass m kg is attached to one end of a light elastic string of natural length 0·5 m and modulus of elasticity $\frac{mg}{2}$ N. The other end of the string is attached to a fixed point O and P hangs vertically below O.
   (a) Find the stretched length of the string when P rests in equilibrium. **(3 marks)**
   (b) Find the elastic potential energy stored in the string in the equilibrium position. **(2 marks)**
   P, which is still attached to the string, is now held at rest at O and then lowered gently into its equilibrium position.
   (c) Find the work done by the weight of the particle as it moves from O to the equilibrium position. **(2 marks)**
   (d) Explain the discrepancy between your answers to parts (b) and (c). **(1 mark)**

4. A particle P, of mass m kg, is attached to two light elastic strings, each of natural length l m and modulus of elasticity $3mg$ N.
The other ends of the strings are attached to the fixed points A and B, where AB is horizontal and $AB = 2l$ m.
If P rests in equilibrium vertically below the mid-point of AB, with each string making an angle θ with the vertical, show that



$$\cot\theta - \cos\theta = \tfrac{1}{6}.$$ **(8 marks)**

5. A small bead P, of mass m kg, can slide on a smooth circular ring, with centre O and radius r m, which is fixed in a vertical plane. P is projected from the lowest point L of the ring with speed $\sqrt{(3gr)}$ ms^{-1}. When P has reached a position such that OP makes an angle θ with the downward vertical, as shown, its speed is v ms^{-1}.



(a) Show that $v^2 = gr(1 + 2\cos\theta)$. **(5 marks)**

(b) Show that the magnitude of the reaction R N of the ring on the bead is given by

$$R = mg(1 + 3\cos\theta).$$ **(4 marks)**

(c) Find the values of $\cos\theta$ when

(i) P is instantaneously at rest, (ii) the reaction R is instantaneously zero. **(2 marks)**

(d) Hence show that the ratio of the heights of P above L in cases (i) and (ii) is 9 : 8. **(3 marks)**

6. A light elastic string, of natural length 0·8 m, has one end fastened to a fixed point O. The other end of the string is attached to a particle P of mass 0·5 kg. When P hangs in equilibrium, the length of the string is 1·5 m.

(a) Calculate the modulus of elasticity of the string. **(3 marks)**

P is displaced to a point 0·5 m vertically below its equilibrium position and released from rest.

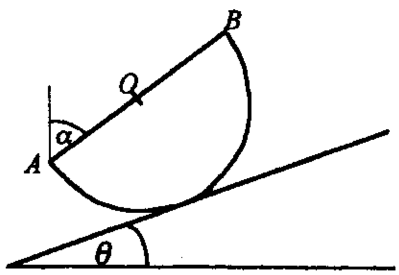
(b) Show that the subsequent motion of P is simple harmonic, with period 1·68 s. **(5 marks)**

(c) Calculate the maximum speed of P during its motion. **(3 marks)**

(d) Show that the time taken for P to first reach a distance 0·25 m from the point of release is 0·28 s, to 2 significant figures. **(4 marks)**

7. (a) Show that the centre of mass of a uniform solid hemisphere of radius r is at a distance $\frac{3r}{8}$ from the centre O of the plane face. **(7 marks)**

The figure shows the vertical cross-section of a rough solid hemisphere at rest on a rough inclined plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{10}$.



(b) Indicate on a copy of the figure the three forces acting on the hemisphere, clearly stating what they are, and paying special attention to their lines of action. **(3 marks)**

(c) Given that the plane face containing the diameter AB makes an angle α with the vertical, show that $\cos\alpha = \frac{4}{5}$. **(6 marks)**