MECHANICS 3 (A) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1. $m\frac{dv}{dt} = -mkv$ $\int \frac{1}{v}\,dv = \int -k\,dt$ $\ln v = -kt + c$ M1 A1 M1 A1
   $\ln u = c$, so $\ln \frac{v}{u} = -kt$ $v = ue^{-kt}$ M1 A1 A1 7

2. $T\cos\theta = mg$, $T\sin\theta = m(0{\cdot}4\sin\theta)\omega^2$ $g = 0{\cdot}4\omega^2\cos\theta$ M1 A1 M1 A1
   $\theta \le 60°$, so $\cos\theta \ge 0{\cdot}5$ $g \ge 0{\cdot}2\omega^2$ $\omega^2 \le 49$ $\omega \le 7$ B1 M1 A1 7

3. (a) $mg = \frac{mge}{2(0{\cdot}5)}$ $e = 0{\cdot}5 \times 2 = 1$ m $OP = 1{\cdot}5$ m M1 A1 A1
   (b) P.E. $= \frac{mg(1^2)}{2(2)(0{\cdot}5)} = \frac{mg}{2}$ J (c) Work done $= mg \times 1{\cdot}5 = \frac{3mg}{2}$ J M1 A1 M1 A1
   (d) Grav. P.E. lost > elastic P.E. gained, because the weight does work in moving the supportive force in (c) B1 8

4. Symmetric, so tensions in strings are equal $2T\cos\theta = mg$ B1 M1 A1
   $AP\sin\theta = l$, so $AP = \frac{l}{\sin\theta}$ $T = \frac{3mg}{l}\left(\frac{l}{\sin\theta} - l\right)$ M1 A1
   Hence $2 \times 3mg\left(\frac{1}{\sin\theta} - 1\right)\cos\theta = mg$ $6(\cot\theta - \cos\theta) = 1$, etc M1 A1 A1 8

5. (a) P.E. gained = K.E. lost : $mgr(1 - \cos\theta) = \frac{1}{2}m(3gr) - \frac{1}{2}mv^2$ M1 M1 A1
   $v^2 = 3gr - 2gr + 2gr\cos\theta = gr(1 + 2\cos\theta)$ M1 A1
   (b) $R - mg\cos\theta = \frac{mv^2}{r}$ M1 A1
   $R = \frac{m}{r}gr(1 + 2\cos\theta) + mg\cos\theta$ $R = mg(1 + 3\cos\theta)$ M1 A1
   (c) (i) When $v = 0$, $\cos\theta = -\frac{1}{2}$ (ii) When $R = 0$, $\cos\theta = -\frac{1}{3}$ B1 B1
   (d) $h_1 = r + \frac{1}{2}r = \frac{3r}{2}$ $h_2 = r + \frac{1}{3}r = \frac{4r}{3}$ Ratio $= \frac{3r}{2} + \frac{4r}{3} = \frac{9}{8}$ M1 A1 A1 14

6. (a) $mg = \frac{\lambda}{0{\cdot}8} \times 0{\cdot}7 = 0{\cdot}5 \times 9{\cdot}8$ $\lambda = 4{\cdot}9 \times \frac{0{\cdot}8}{0{\cdot}7} = 5{\cdot}6$ N M1 A1 A1
   (b) $(0{\cdot}5 \times 9{\cdot}8) - \frac{5{\cdot}6}{0{\cdot}8}(0{\cdot}7 + x) = 0{\cdot}5\ddot{x}$ $4{\cdot}9 - 4{\cdot}9 - 7x = 0{\cdot}5\ddot{x}$ M1 A1
   $\ddot{x} = -14x$, of form $\ddot{x} = n^2x$ with $n^2 = 14$, so simple harmonic A1 A1
   Period $= 2\pi/\sqrt{14} = 1{\cdot}68$ s A1
   (c) Maximum speed $= an = 0{\cdot}5\sqrt{14} = 1{\cdot}87$ ms^{-1} B1 M1 A1
   (d) $x = 0{\cdot}5\cos nt$ $0{\cdot}25 = 0{\cdot}5\cos nt$ $\cos nt = 0{\cdot}5$ B1 M1
   $nt = \frac{\pi}{3}$ $t = 0{\cdot}28$ s A1 A1 15

7. (a) $x^2 + y^2 = r^2$ $\bar{x}\int_0^r \pi y^2\,dx = \int_0^r \pi xy^2\,dx$ B1 M1 A1
   $\pi\bar{x}\int_0^r r^2 - x^2\,dx = \pi\int_0^r r^2x - x^3\,dx$ $\frac{2r^2}{3}\bar{x} = \frac{r^4}{4}$ $\bar{x} = \frac{3r}{8}$ M1 A1 A1 A1
   (b) Forces shown: weight, normal reaction (thro' O), friction at plane B1 B1 B1
   (c) Resolve // plane : $F = W\sin\alpha$ M(O) : $Fr = W\frac{3r}{8}\cos\alpha$ B1 M1 A1
   $Wr\sin\theta = \frac{3r}{8}W\cos\alpha$ $\cos\alpha = \frac{8}{3} \times \frac{3}{10} = \frac{4}{5}$ M1 A1 A1 16