Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Gold Level G4

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
55	45	28	22	16	10

1. Two uniform rods AB and BC are rigidly joined at B so that $\angle ABC = 90°$. Rod AB has length 0.5 m and mass 2 kg. Rod BC has length 2 m and mass 3 kg. The centre of mass of the framework of the two rods is at G.

(*a*) Find the distance of G from BC.

(2)

The distance of G from AB is 0.6 m.
The framework is suspended from A and hangs freely in equilibrium.

(*b*) Find the angle between AB and the downward vertical at A.

(3)

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2. A particle P of mass 0.6 kg is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at 30° to the horizontal. When P has moved 12 m, its speed is 4 m s^{-1}. Given that friction is the only non-gravitational resistive force acting on P, find

(*a*) the work done against friction as the speed of P increases from 0 m s^{-1} to 4 m s^{-1},

(4)

(*b*) the coefficient of friction between the particle and the plane.

(4)

3.

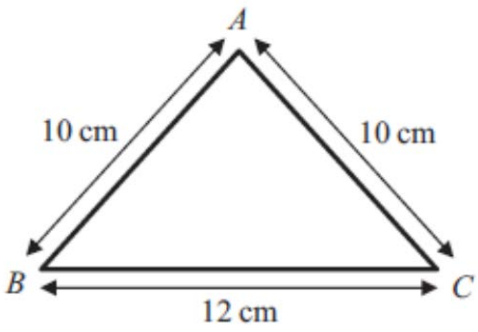

Figure 1

A triangular frame is formed by cutting a uniform rod into 3 pieces which are then joined to form a triangle ABC, where $AB = AC = 10$ cm and $BC = 12$ cm, as shown in Figure 1.

(*a*) Find the distance of the centre of mass of the frame from BC.

(5)

The frame has total mass M. A particle of mass M is attached to the frame at the mid-point of BC. The frame is then freely suspended from B and hangs in equilibrium.

(*b*) Find the size of the angle between BC and the vertical.

(4)

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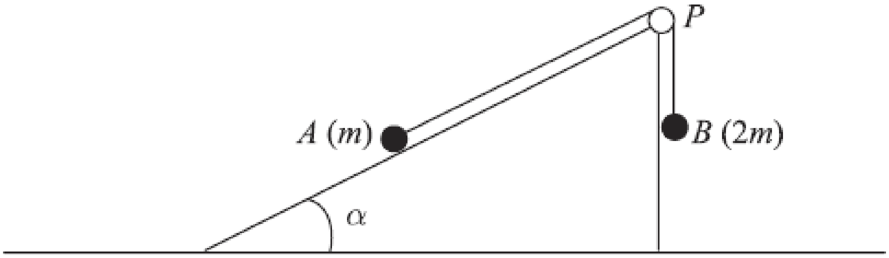
4. **Figure 2**



Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The string passes over a small light smooth pulley P fixed at the top of the plane. The particle B hangs freely below P, as shown in Figure 2. The particles are released from rest with the string taut and the section of the string from A to P parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{5}{8}$. When each particle has moved a distance h, B has not reached the ground and A has not reached P.

(*a*) Find an expression for the potential energy lost by the system when each particle has moved a distance h.

(2)

When each particle has moved a distance h, they are moving with speed v. Using the work-energy principle,

(*b*) find an expression for v^2, giving your answer in the form kgh, where k is a number.

(5)

5.

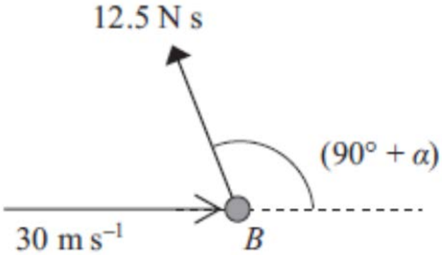


Figure 3

A small ball B of mass 0.25 kg is moving in a straight line with speed 30 m s^{-1} on a smooth horizontal plane when it is given an impulse. The impulse has magnitude 12.5 N s and is applied in a horizontal direction making an angle of $(90° + \alpha)$, where $\tan \alpha = \frac{3}{4}$, with the initial direction of motion of the ball, as shown in Figure 3.

(i) Find the speed of B immediately after the impulse is applied.

(ii) Find the direction of motion of B immediately after the impulse is applied.

(6)

6.

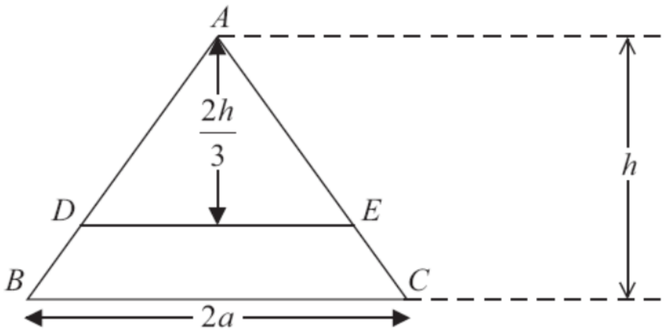


Figure 2

A uniform triangular lamina *ABC* of mass *M* is such that *AB* = *AC*, *BC* = 2*a* and the distance of *A* from *BC* is *h*. A line, parallel to *BC* and at a distance $\frac{2h}{3}$ from *A*, cuts *AB* at *D* and cuts *AC* at *E*, as shown in Figure 2.

It is given that the mass of the trapezium *BCED* is $\frac{5M}{9}$.

(*a*) Show that the centre of mass of the trapezium *BCED* is $\frac{7h}{45}$ from *BC*.

(5)

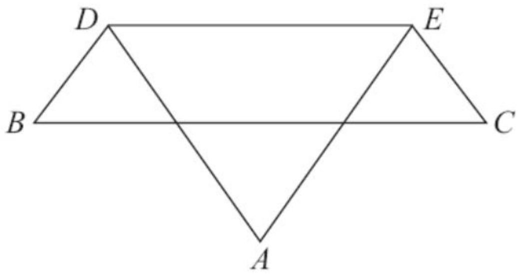


Figure 3

The portion *ADE* of the lamina is folded through 180° about *DE* to form the folded lamina shown in Figure 3.

(*b*) Find the distance of the centre of mass of the folded lamina from *BC*.

(4)

The folded lamina is freely suspended from *D* and hangs in equilibrium. The angle between *DE* and the downward vertical is α.

(*c*) Find tan α in terms of *a* and *h*.

(4)

7.

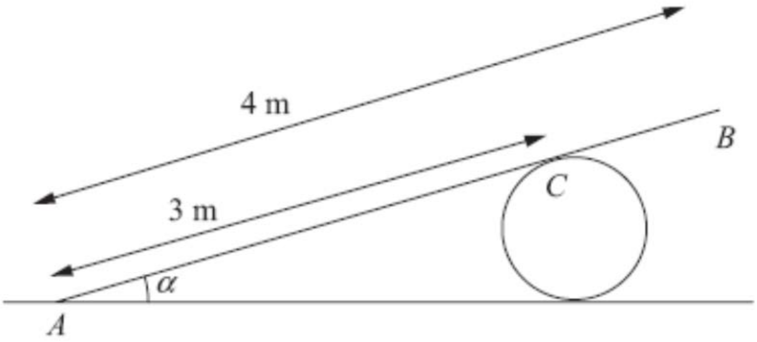


Figure 4

A uniform plank *AB*, of weight 100 N and length 4 m, rests in equilibrium with the end *A* on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is *C*, where *AC* = 3 m, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle α to the horizontal, where $\sin \alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is μ.

Modelling the plank as a rod, find the least possible value of μ.

(10)

8. [*In this question* **i** *and* **j** *are unit vectors in a horizontal and upward vertical direction respectively*.]

A particle P is projected from a fixed point O on horizontal ground with velocity $u(\mathbf{i} + c\mathbf{j})$ m s^{-1}, where c and u are positive constants. The particle moves freely under gravity until it strikes the ground at A, where it immediately comes to rest. Relative to O, the position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})$ m.

(*a*) Show that

$$y = cx - \frac{4.9x^2}{u^2}.$$

(5)

Given that $u = 7$, $OA = R$ m and the maximum vertical height of P above the ground is H m,

(*b*) using the result in part (*a*), or otherwise, find, in terms of c,

(i) R

(ii) H.

(6)

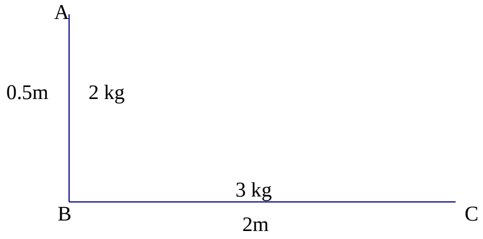
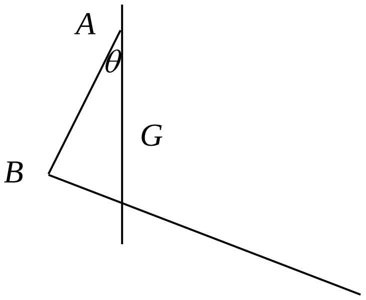
Given also that when P is at the point Q, the velocity of P is at right angles to its initial velocity,

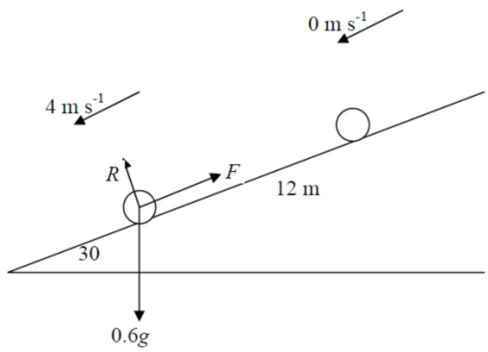
(*c*) find, in terms of c, the value of x at Q.

(6)

TOTAL FOR PAPER: 75 MARKS

END

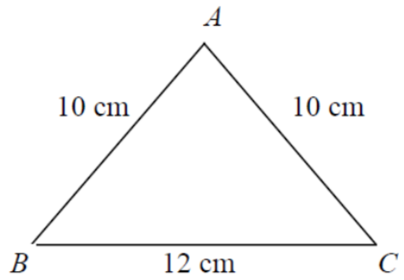
Question Number	Scheme	Marks
1. (a)		M1
	$5\bar{y} = 2 \times 0.25(+0)$	A1
	$\bar{y} = \dfrac{2 \times 0.25}{5} = 0.1$	
(b)		M1
	$\tan\theta = \dfrac{0.6}{0.5 - 0.1}$	A1ft
	$\theta = \tan^{-1}\left(\dfrac{6}{4}\right) = 56.3° = 56°$	A1

Question Number	Scheme	Marks
2.		
(a)	K.E gained = $\frac{1}{2} \times 0.6 \times 4^2$ P.E. lost = $0.6 \times g \times (12\sin 30)$ Change in energy = P.E. lost – K.E. gained $= 0.6 \times g \times 12\sin 30 - \frac{1}{2} \times 0.6 \times 4^2$ = 30.48 Work done against friction= 30 or 30.5 J	 **M1 A1 A1** **A1** **(4)**
(b)	R $(\uparrow)$ $R = 0.6g\cos 30$ $F = \frac{30.48}{12}$ $F = \mu R$ $\mu = \frac{30.48}{12 \times 0.6g\cos 30}$ $\mu = 0.4987$ $\mu = 0.499$ or 0.50	**B1** **B1ft** **M1** **A1** **(4)** **[8]**

Question Number	Scheme	Marks

3

(a)



	AB	AC	BC	frame
mass ratio	10	10	12	32
dist from BC	4	4	0	$\bar{x}$

B1
B1

Moments about BC:

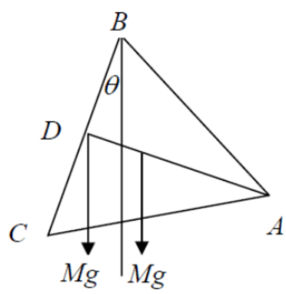
$$10 \times 4 + 10 \times 4 + 0 = 32\bar{x}$$ **M1 A1**

$$\bar{x} = \frac{80}{32}$$

$$\bar{x} = 2\frac{1}{2}\ (2.5)$$ **A1**

(5)

(b)



Moments about B: **M1 A1 A1**

$Mg \times 6\sin\theta = Mg \times \left(x\cos\theta - 6\sin\theta\right)$

$12\sin\theta = x\cos\theta$

$\tan\theta - \dfrac{\bar{x}}{12}$

$\theta = 11.768..... = 11.8^{o}$ **A1**

Alternative method: **(4)**

C of M of loaded frame at distance $\frac{1}{2}\bar{x}$ from D along DA. **B1**

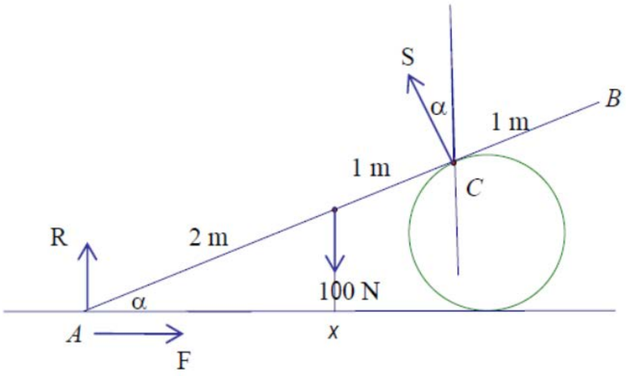
$$\tan\theta = \frac{\frac{1}{2}\bar{x}}{6}$$ **M1 A1**

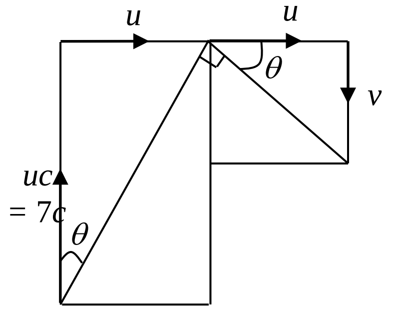
$$\theta = 11.768..... = 11.8^{o}$$ **A1**

[9]

Question Number	Scheme	Marks
4. (a)	PE lost = $2mgh - mgh \sin \alpha$ (= $7mgh/5$)	M1 A1
(b)	Normal reaction $R = mg \cos \alpha$ (= $4mg/5$)	B1
	Work-energy: $\frac{1}{2}mv^2 + \frac{1}{2}.2mv^2 = \frac{7mgh}{5} - \frac{5}{8}.\frac{4mg}{5}.h$	M1 A2,1,0
	$\Rightarrow \frac{3}{2}mv^2 = \frac{9mgh}{10} \Rightarrow v^2 = \frac{3}{5}gh$	A1
5.	$12.5\sin\alpha = \frac{1}{4}(v_1 - -30)$	M1
	or $-12.5\sin\alpha = \frac{1}{4}(v_1 - 30)$ $(v_1 = 0)$	A1
	$12.5\cos\alpha = \frac{1}{4}(v_2 - 0)$ $(v_2 = 40)$	M1 A1
	speed is 40 m s^{-1};	A1
	perpendicular to original direction	A1
		6
OR	Using a vector triangle: $(\frac{1}{4}v)^2 = 7.5^2 + 12.5^2 - 2\text{x}7.5\text{x}12.5\cos(90^\circ - \alpha)$	M1 A1
	$v = 40\text{m s}^{-1}$	A1
	$\frac{12.5}{\sin\theta} = \frac{7.5}{\sin\alpha}$	M1 A1
	$\theta = 90^\circ$	A1
		6

Question Number	Scheme	Marks
6. **(a)**	*ABC* *ADE* *BCED*	
	M $\frac{4M}{9}$ $\frac{5M}{9}$	B1
	$\frac{h}{3}$ $(\frac{h}{3}+\frac{1}{3}\frac{2h}{3})$ $\bar{y}$	B1
		M1
	$M\frac{h}{3}-\frac{4M}{9}\ \frac{5h}{9}=\frac{5M}{9}\bar{y}$	A1
	$\bar{y}=\frac{7h}{45}$ ***Answer Given***	A1
		(5)
(b)		M1
	$\frac{5M}{9}\frac{7h}{45}+\frac{4M}{9}\left(\frac{h}{3}-\frac{1}{3}\times\frac{2h}{3}\right)=M\ \bar{x}$	A1 A1
	$\bar{x}=\frac{11h}{81}$	A1
		(4)
(c)	$\tan\alpha=\dfrac{\frac{h}{3}-\bar{x}}{\frac{2a}{3}}$	M1
		A1 ft
	$=\frac{8h}{27a}$	DM1 A1
		(4) **[13]**

Question Number	Scheme	Marks
7.		
	Taking moments about A:	
	$3S = 100 \times 2 \times \cos\alpha$	**M1 A1**
	Resolving vertically:	
	$R + S\cos\alpha = 100$	**M1 A1**
	Resolving horizontally: $S\sin\alpha = F$	**M1 A1**
	(Most alternative methods need 3 independent equations, each one worth M1A1. Can be done in 2 e.g. if they resolve horizontally and take moments about X then $R \times 2\times \cos\alpha = S \times (3 - 2\times \cos^2\alpha)$ scores M2A2)	
	Substitute trig values to obtain correct values for F and R (exact or decimal equivalent).	**DM1**
	$\left(S = \frac{200\sqrt{8}}{9}\right), R = 100 - \frac{1600}{27} = \frac{1100}{27} \approx 40.74, F = \frac{200\sqrt{8}}{27} \approx 20.95 \ldots$	**A1**
	$F \le \mu R$, $200\sqrt{8} \le \mu \times 1100$, $\mu \ge \frac{200\sqrt{8}}{1100} = \frac{2\sqrt{8}}{11}$	**M1**
	Least possible μ is 0.514 (3sf), or exact.	**A1** **[10]**

Question Number	Scheme	Marks
8. **(a)**	$x = ut$	**B1**
	$y = cut - 4.9t^2$	**M1 A1**
	eliminating t and simplifying to give $y = cx - \dfrac{4.9x^2}{u^2}$ **	**DM1 A1** **(5)**
(b)(i)	$0 = cx - \dfrac{4.9x^2}{u^2}$	**M1**
	$0 = x(c - \dfrac{4.9x}{u^2}) \Rightarrow R = \dfrac{u^2c}{4.9} = 10c$	**M1 A1**
(ii)	When $x = 5c$, $y = H$	**M1**
	$= 5c^2 - \dfrac{(5c\ )^2}{10} = 2.5c^2$	**M1 A1** **(6)**
(c)	$\dfrac{dy}{dx} = c - \dfrac{9.8x}{u^2} = c - \dfrac{x}{5}$	**M1 A1**
	When $x = 0$, $\dfrac{dy}{dx} = c$	**B1**
	So, $c - \dfrac{x}{5} = \dfrac{-1}{c}$	**DM1 A1**
	$x = 5(c + \dfrac{1}{c})$	**A1** **(6)**
		[17]
	Alternative to 8(c)	
	 $\tan\theta = \dfrac{u}{cu} = \dfrac{1}{c} = \dfrac{v}{u}$	**B1**
	$\Rightarrow v = \dfrac{u}{c} = \dfrac{7}{c}$	**M1 A1**
	$v = u + at$; $-\dfrac{7}{c} = 7c - 9.8t$	**M1**
	$t = \dfrac{7}{9.8}(c + \dfrac{1}{c})$	**A1**
	$x = ut = 7t$; $x = 5(c + \dfrac{1}{c})$	**A1**

Examiner reports

Question 1

Q1(a) was an accessible question about centre of mass and many correct answers were seen. However, the fact that the ratio of the masses of the two rods was not the same as the ratio of their lengths caused some difficulties and a significant number of candidates did not have the centres of mass of the rods at the centres of the rods.

Some candidates took moments about an axis through A parallel to BC, but then forgot to find the distance asked for in the question. The question only asked candidates to find the distance of the centre of mass from BC, but many candidates also found the distance from AB.

In Q1(b) many candidates used the correct triangle and trig ratio to find the angle. Candidates should be reminded that when a value is given in the question they are expected to use it; some candidates who had obtained an answer other than 0.6 for the distance of the centre of mass from AB used their own incorrect value. Some candidates found it hard to determine the correct triangle to work with, often as a result of a poor sketch or no sketch at all.

Question 2

This question proved to be straightforward for well-prepared candidates.

In part (a) it was pleasing to see many candidates tackling this using the work-energy method, and there was less evidence this year of candidates double counting by including both the change in GPE and the work done against the weight, but candidates sometimes confused work done with just potential energy lost, or just kinetic energy gained. The alternative method using *suvat* to find the acceleration and then using $F = m$a was also common. In the final answers there was considerable confusion between work done against friction and the frictional force. Many lost the final A mark by leaving the answer as 30.48 despite having used $g = 9.8$.

In part (b) candidates frequently did not make the connection with part (a) and proceeded to start again from scratch. In this case, a common but expensive error was to omit the component of the weight from their equation of motion.

Question 3

Some candidates struggled with this question. Despite the question being explained clearly with reference to rods it was not uncommon to see the triangle treated as a lamina. Another common error was to treat the rods as being of equal mass.

The geometry of the symmetrical triangular figure was appreciated by nearly all candidates with the height of the triangle correctly calculated as 8 cm, although it was disappointing to find several candidates not recognising the 3-4-5 triangle and engaging in more work than expected to find the height of the triangle.

For part (a) those candidates who answered the question as set and worked with three rods had little difficulty in producing a relevant moments equation and arriving at the correct result. However it was disappointing to find a significant number of candidates treating the triangle as a lamina, and they were happy simply to write down the answer as $\frac{8}{3}$ cm.

For part (b) most candidates could either write down or calculate the distance of the new centre of mass from *BC* and proceed to find the required angle. 3 out of 4 marks were available for those who had treated the shape as a lamina. A number of candidates ignored the extra particle added to the framework and answered their own question. Very few students used the method of taking moments about *B* to find the angle.

Question 4

Many candidates lost several marks on this question. Some simply did not attempt the question, other presented confident, but incorrect working.

Part (a) Many errors were made; some were simply a case of the ambiguous answer "loss of GPE = $-\frac{7mgh}{5}$", but it was also common to see both particles regarded as losing GPE, or the assumption that both particles move a vertical distance h.

Part (b) Some candidates clearly did not want to attempt this using work and energy. Those who did often tried to look at each particle separately rather than consider the system as a whole, and often ran into difficulties, double counting some elements. The normal reaction was usually identified correctly, leading to a correct expression for the work done against the frictional force. Two particularly common errors were the omission of the kinetic energy of *B* (giving an equation with $\frac{1}{2}mv^2$ rather than $\frac{3}{2}mv^2$), and double counting the increase in GPE for *A*.

Question 5

Despite being on a familiar topic, this question was not answered well, possibly because the candidates are used to seeing it presented in vector form. It was common to see candidates trying to use the impulse of 12.5 in an impulse-momentum equation without understanding that it is a vector equation. Many candidates offered the incorrect equation $12.5 = 0.25(v - 30)$ leading to $v = 80$.

The most successful approach was either to adopt vector notation or to deal with the horizontal and vertical components separately. There was some trig confusion in finding the components of the impulse, and there were sign errors in the component parallel to the initial direction. The question does ask for the speed of *B* after the collision, so answers left as vectors did not earn this mark. Very few candidates adopted the vector triangle approach.

Question 6

(a) The majority of candidates saw this as a large triangle with a small triangle removed. Despite being told that the mass of the trapezium was $\frac{5M}{9}$ some candidates struggled to find the mass of the small triangle. Nearly all used the position of the centre of mass of a triangle correctly, and the given answer was often reached correctly. A popular alternative approach was to split the trapezium in to a rectangle and two identical right angled triangles.

(b) Despite their success with part (a) many candidates struggled to combine the two shapes in an alternative formation. A common error was for the masses of the two pieces not to add up to the mass of the original triangle.

(c) Most candidates were able to identify the required triangle, but it was common to see errors in the length of $\frac{1}{2}DE$.

Question 7

Although there were many fully correct responses to this question, the unstructured nature of the problem did present difficulties for some candidates. A clearly labelled diagram showing all the forces was essential. Some candidates were unsure of the direction of action of the normal reactions at *A* and at *C*. Some gave them both the same name, and appeared to believe that they were equal in magnitude. Others omitted at least one of the normal reactions. Some candidates used horizontal and vertical components for the force at *C* but were usually unable to connect them later in their solution. Many candidates recognised the need to take moments and to resolve but errors were often made in doing so. The most straightforward approach of taking moments about *A* and then resolving vertically and horizontally was often seen. Many candidates took moments about *C* or tried to resolve parallel to and perpendicular to the rod, but this frequently resulted in a missing term. There were a small number of more imaginative solutions involving moments about points not on the plank. Although there was evidence of confusion between sine and cosine when resolving forces, incorrect solutions usually involved an attempt to resolve when it was not necessary, or a failure to do so when it was required. Equally, it was common to find distances missing from a moments equation. Despite having correct equations many candidates could not combine them to find friction and reaction forces correctly. Some candidates did demonstrate that they were finding the least possible value of μ, but many did not address this point and used $F = \mu R$ throughout. A few candidates misread the question, using 100 g as the weight.

Question 8

(a) Many correct solutions were seen, with the majority of candidates clearly familiar with the method for deriving the equation of the parabolic path. However some candidates substituted into all the *suvat* equations and were clearly struggling to find a way forward.

(b) Those candidates who were not able to complete (a) started afresh at this point, and were often successful in earning marks in this part. Most candidates used the given formula to find R having identified this as the value of x when $y = 0$. Some candidates used the fact that maximum height occurs when $x = \frac{R}{2}$, but many preferred to work with the initial information and to use $v^2 = u^2 + 2as$ to find H.

(c) Only a few candidates made a concerted attempt at this part with a correct solution a rarity. Most attempts used a vector approach rather than calculus. Several candidates demonstrated an understanding of perpendicular vectors, and the partially correct velocity $\begin{pmatrix} cu \\ -u \end{pmatrix}$ in place of the correct answer $\begin{pmatrix} u \\ \frac{-u}{c} \end{pmatrix}$ was quite common. Diagrams were rarely seen, possibly accounting for the common incorrect answer $\begin{pmatrix} u \\ -cu \end{pmatrix}$. Confusion between distances and velocities often marred any work beyond this stage.

Statistics for M2 Practice Paper Gold 4

				Mean average scored by candidates achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	5	5	75.0	3.75	4.53	4.25	3.47	2.60	1.64	1.61	0.64
2	8		64.9	5.19	6.69	5.94	4.92	4.23	3.27	2.67	1.80
3	9		49.9	4.49	6.98	5.59	3.79	2.85	1.98	1.42	0.88
4	7		30.6	2.14		3.33	1.76	1.15	0.72	0.48	0.23
5	6		42.7	2.56	4.26	3.36	1.92	1.30	0.95	0.58	0.51
6	13		46.6	6.06	9.46	6.81	4.43	4.51	3.33	2.40	1.04
7	10		58.7	5.87	7.88	6.80	4.25	2.72	2.14	0.74	0.24
8	17		39.3	6.68		8.66	3.74	2.27	1.67	0.95	0.95
	75		**49.0**	**36.74**		**44.74**	**28.28**	**21.63**	**15.70**	**10.85**	**6.29**