Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Gold Level G3

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
57	**47**	**35**	**29**	**23**	**17**

1. A particle of mass 0.25 kg is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ m s^{-1} when it receives the impulse $(5\mathbf{i} - 3\mathbf{j})$ N s.

Find the speed of the particle immediately after the impulse.

(5)

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2. A particle P of mass 3 kg moves from point A to point B up a line of greatest slope of a fixed rough plane. The plane is inclined at 20° to the horizontal. The coefficient of friction between P and the plane is 0.4.

Given that AB = 15 m and that the speed of P at A is 20 m s^{-1}, find

(*a*) the work done against friction as P moves from A to B,

(3)

(*b*) the speed of P at B.

(4)

3.

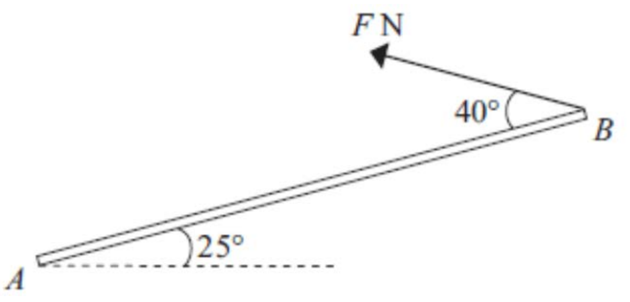

Figure 1

A uniform rod AB, of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25° above the horizontal by a force of magnitude F newtons applied to its end B. The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40° with the rod, as shown in Figure 1.

(*a*) Find the value of F.

(4)

(*b*) Find the magnitude and direction of the vertical component of the force acting on the rod at A.

(4)

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4. A rough circular cylinder of radius $4a$ is fixed to a rough horizontal plane with its axis horizontal. A uniform rod AB, of weight W and length $6a\sqrt{3}$, rests with its lower end A on the plane and a point C of the rod against the cylinder. The vertical plane through the rod is perpendicular to the axis of the cylinder. The rod is inclined at 60° to the horizontal, as shown in Figure 1.

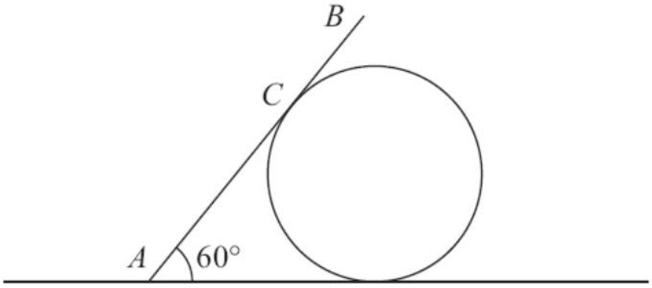

Figure 1

(a) Show that $AC = 4a\sqrt{3}$.

(2)

The coefficient of friction between the rod and the cylinder is $\frac{\sqrt{3}}{3}$ and the coefficient of friction between the rod and the plane is μ. Given that friction is limiting at both A and C,

(b) find the value of μ.

(9)

5.

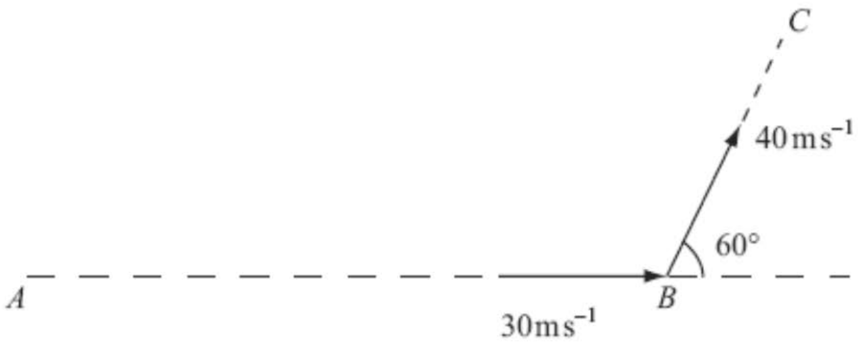

Figure 1

The points *A*, *B* and *C* lie in a horizontal plane. A batsman strikes a ball of mass 0.25 kg. Immediately before being struck, the ball is moving along the horizontal line *AB* with speed 30 m s^{-1}. Immediately after being struck, the ball moves along the horizontal line *BC* with speed 40 m s^{-1}. The line *BC* makes an angle of 60° with the original direction of motion *AB*, as shown in Figure 1.

Find, to 3 significant figures,

(i) the magnitude of the impulse given to the ball,

(ii) the size of the angle that the direction of this impulse makes with the original direction of motion *AB*.

(8)

6.

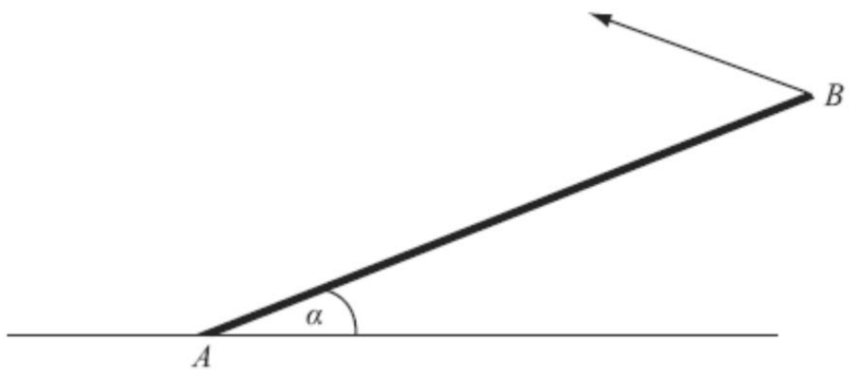

Figure 2

A uniform rod *AB*, of mass 20 kg and length 4 m, rests with one end *A* on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$, by a force acting at *B*, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5.

Find the magnitude of the normal reaction of the ground on the rod at *A*.

(7)

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7. Two small spheres P and Q of equal radius have masses m and $5m$ respectively. They lie on a smooth horizontal table. Sphere P is moving with speed u when it collides directly with sphere Q which is at rest. The coefficient of restitution between the spheres is e, where $e > \frac{1}{5}$.

(*a*) (i) Show that the speed of P immediately after the collision is $\frac{u}{6}(5e - 1)$.

(ii) Find an expression for the speed of Q immediately after the collision, giving your answer in the form λu, where λ is in terms of e.

(6)

Three small spheres A, B and C of equal radius lie at rest in a straight line on a smooth horizontal table, with B between A and C. The spheres A and C each have mass $5m$, and the mass of B is m. Sphere B is projected towards C with speed u. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.

(*b*) Show that, after B and C have collided, there is a collision between B and A.

(3)

(*c*) Determine whether, after B and A have collided, there is a further collision between B and C.

(4)

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8. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where v is given by

$$v = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \le t \le 4 \\ 16 - 2t, & t > 4. \end{cases}$$

When $t = 0$, P is at the origin O.

Find

(*a*) the greatest speed of P in the interval $0 \le t \le 4$,

(4)

(*b*) the distance of P from O when $t = 4$,

(3)

(*c*) the time at which P is instantaneously at rest for $t > 4$,

(1)

(*d*) the total distance travelled by P in the first 10 s of its motion.

(8)

TOTAL FOR PAPER: 75 MARKS

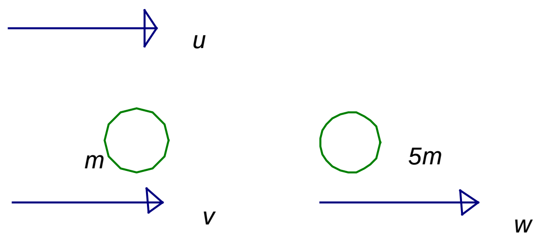
END

Question Number	Scheme	Marks
1.	**I** $= m\mathbf{v} - m\mathbf{u}$ $5\mathbf{i} - 3\mathbf{j} = \frac{1}{4}\mathbf{v} - \frac{1}{4}(3\mathbf{i} + 7\mathbf{j})$ $\mathbf{v} = 23\mathbf{i} - 5\mathbf{j}$ $\lvert\mathbf{v}\rvert = \sqrt{23^2 + 5^2} = 23.5$	**M1A1** **A1** **M1A1** **[5]**
2 **(a)**	Work done = $15\mu R = 15 \times 0.4 \times 3g \cos 20°$ $= 18g \cos 20 = 166$ (J)	**M1** **M1** **A1** **(3)**
(b)	Energy: WD against F + GPE + final KE = initial KE $\text{their WD} + 3g \sin 20° \times 15 + \frac{1}{2}3v^2 = \frac{1}{2}3 \times 20^2$ $v = 13.7$ (m s^{-1})	**M1A2ft** **A1** **(4)** **[7]**
3 **(a)**	$M(A)$, $F.\ 4\sin 40° = 5g.2\cos 25°$ $F = 35$	**M1** **A1** **A1** **A1** **(4)**
(b)	$F \cos 75° \pm Y = 5g$ $Y = 40$; UP	**M1** **A1** **A1** **A1** **(4)** **[8]**

Question Number	Scheme	Marks
4.		
(a)	$AC = 4a\tan 60^o = 4a\sqrt{3}.$	**M1 A1**
		(2)
(b)	use of $F = \mu R$ at either A or C	**M1**
	3 independent equations required. Award M1A1 for each in the order seen. If more than 3 relevant equations seen, award the marks for the best 3.	
	$M(A)$, $R_C.4a\sqrt{3} = W.3a\sqrt{3}\cos 60^o$	**M1 A1**
	$(\uparrow)$, $R_A + R_C\cos 60^o + F_C\cos 30^o = W$	**M1 A1**
	$(\rightarrow)$, $F_A - R_C\cos 30^o + F_C\cos 60^o = 0$	**M1 A1**
	M(C) $a\sqrt{3}\cos 60W + F_A.4a\sqrt{3}\sin 60 = R_A.4a\sqrt{3}\cos 60$	
	Parallel: $F_A\cos 60 + R_A\cos 30 + F_C = W\cos 30$	
	Perpendicular: $R_C + R_A\cos 60 = F_A\cos 30 + W\cos 60$	
	solving to give $\mu = \frac{\sqrt{3}}{5}$; 0.346 or 0.35.	**DM1** **A1**
	Reactions in the wrong direction(s) – check carefully	
		(9) **[11]**

Question Number	Scheme	Marks
5.	(i) $I\uparrow = 0.25\times 40\sin 60 = 5\sqrt{3}$ (8.66) one component $I\leftarrow = 0.25(-20+30) = 2.5$ both $\lvert I \rvert = \sqrt{75+6.25} = 9.01$ (Ns)	**M1** **A1** **M1 A1** **(4)**
	(ii) $\frac{\sin\theta}{40} = \frac{\sin 60^o}{\sqrt{1300}}$ $\theta = 106^o$ (3 s.f.)	**M1 A1**
	or $\tan\theta = \pm\frac{5\sqrt{3}}{2.5}$ oee $\theta = 106°$	**M1 A1** **(4)**
		[8]

Question Number	Scheme	Marks
6.	$m(B): R\times 4\cos\alpha = F\times 4\sin\alpha + 20g\times 2\cos\alpha$	M1 A2
	Use of $F = \frac{1}{2}R$	M1
	Use of correct trig ratios	B1
	R = 160N or 157N	DM1 A1
		[7]

Question Number	Scheme	Marks
7.		
(a)	CLM: $mv + 5mw = mu$ NLI: $w - v = eu$	**B1** **B1**
	Solve v: $v = \frac{1}{6}(1-5e)u$, so speed = $\frac{1}{6}(5e-1)u$ (**NB – answer given on paper**)	**M1* A1**
	Solve w: $w = \frac{1}{6}(1+e)u$	**M1* A1**
	* The M's are dependent on having equations (not necessarily correct) for CLM and NLI	**(7)**
(b)	After B hits C, velocity of B = "v" = $\frac{1}{6}(1-5.\frac{4}{5})u = -\frac{1}{2}u$	**M1 A1**
	velocity < 0 $\Rightarrow$ change of direction $\Rightarrow$ B hits A	**A1 CSO** **(3)**
(c)	velocity of C after = $\frac{3}{10}u$	**B1**
	When B hits A, "u" = $\frac{1}{2}u$, so velocity of B after = $-\frac{1}{2}(-\frac{1}{2}u) = \frac{1}{4}u$	**B1**
	Travelling in the same direction but $\frac{1}{4} < \frac{3}{10}$ $\Rightarrow$ <u>no second collision</u>	**M1** **A1 CSO** **(4)**

8.		
(a)	$0 \le t \le 4$: $a = 8 - 3t$ $a = 0 \Rightarrow t = 8/3$ s $\rightarrow v = 8.\frac{8}{3} - \frac{3}{2}.\left(\frac{8}{3}\right)^2 = \frac{32}{3}$ (m/s) second M1 dependent on the first, and third dependent on the second.	**M1** **DM1** **DM1 A1** **(4)**
(b)	$s = 4t^2 - t^3/2$ $t = 4$: $s = 64 - 64/2 = \underline{32 \text{ m}}$	**M1** **M1 A1** **(3)**
(c)	$t > 4$: $v = 0 \Rightarrow t = \underline{8 \text{ s}}$	**B1 (1)**
(d)	*Either* $t > 4$ $s = 16t - t^2$ $(+ C)$ $t = 4, s = 32 \rightarrow C = -16 \Rightarrow s = 16t - t^2 - 16$ $t = 10 \rightarrow s = 44$ m But direction changed, so: $t = 8, s = 48$ Hence total dist travelled = 48 + 4 = <u>52 m</u> *Or (probably accompanied by a sketch?)* t=4 v=8, t=8 v=0, so area under line = $\frac{1}{2} \times (8-4) \times 8$ t=8 v=0, t=10 v=-4, so area above line = $\frac{1}{2} \times (10-8) \times 4$ total distance = 32(from b) + 16 + 4 = <u>52 m.</u> Or M1, A1 for t > 4 $\frac{dv}{dt} = -2$, =constant t=4, v=8; t=8, v=0; t=10, v=-4 M1, A1 $s = \frac{u+v}{2}t = \frac{32}{2}t$, =16 working for t = 4 to t = 8 M1, A1 $s = \frac{u+v}{2}t = \frac{-4}{2}t$, =-4 working for t = 8 to t = 10 M1, A1 total = 32+14+4, =52	**M1** **M1 A1** **M1 A1** **M1** **DM1 A1** **M1A1A1** **M1A1A1** **M1A1** **(8)**

Examiner reports

Question 1

Candidates found this very accessible with the majority obtaining the correct velocity. Unfortunately many did not proceed to find the speed, which was a careless loss of two marks. Common errors included sign errors in the original equation, or in rearranging the equation, and errors in manipulating the fractions. Some candidates made the mistake of trying to work with the magnitudes of impulse and momentum.

Question 2

This proved to be a straightforward question for the majority of candidates.

In part (a) the majority of candidates were able to resolve correctly, and almost all understood that finding the work done involved multiplying force by distance. The question was very specific in asking for the work done against the friction, and too many candidates thought that this needed to include the work done against the weight. It was common to see the final answer given to an inappropriate level of accuracy.

The majority of candidates attempted part (b) by forming a work/energy equation. Most attempts included all of the required terms, but there were frequently sign errors, either in placing the work done against friction or in the change in kinetic energy. There are still many candidates including both the work done against the weight and the change in gravitational potential energy, not recognising that this is the same thing. Some energy equations did not include the work done against friction at all.

Candidates using the alternative approach via the *suvat* formulae often muddled the signs in their equations. Several did not realise that the acceleration up the plane was actually a deceleration.

Question 3

In part (a) many students found the value of F correctly, usually by following the most direct method of taking moments about A. There were a few errors with sin/cos confusion and some candidates omitted one or more lengths from their moment's equation.

For part (b), having used moments in part (a), many candidates tried the same approach here. The majority tried to take moments about B, but they usually omitted the horizontal component of the force acting on the rod at A. The other common approach was to resolve vertically, but this also proved difficult because many candidates considered the component of F perpendicular to the rod rather than the vertical component of F. Some candidates did give completely correct answers to this part, but many were confused by the simplicity of what they were being asked and preferred to give the magnitude and direction of the force acting at A, rather than work on just the vertical component of that force.

Question 4

(a) Some candidates did not give clear explanations to justify why $AC = 4a\tan 60$, usually because they did not identify the right angle triangle formed by using the centre of the circle on the diagram given.

(b) Candidates needed to form and use three separate equations by resolving or taking moments. In the better solutions these equations were clearly labelled, but it was often necessary to guess what the candidate was trying to do. A clearly labelled diagram showing the labels and the directions of the forces acting helped to avoid errors. Some candidates formed sufficient equations but could not find a way to use their equations to find the value of μ. Although it is inconceivable that the rod might slip upwards and to the right, several candidates did have friction acting in the wrong direction, and friction did not always seem to be acting parallel to the direction of motion if the rod were to slip.

Question 5

Many candidates did not recognise this as a question on the impulse-momentum principle in vector form. Many of the weaker candidates simply worked with the given magnitudes. Some realised the need to resolve, but resolved and used only the component in the initial direction. Those candidates who resolved correctly had no problems with finding the magnitude of the impulse, though some left their answer as a vector.

Most candidates with an impulse (or change in velocity) in component form went on to find an angle. Unfortunately the majority of them found the supplementary angle, the angle to *BA* instead of *AB*, often without reference to a diagram with a marked angle.

Some candidates who struggled to find the impulse made a fresh start to find the angle, often drawing a correct vector triangle and using trigonometry to find the correct angle (or its supplement) without realising that the same diagram could have helped them with the impulse.

Question 6

There were very few correct solutions to this question that did not involve taking moments about *B*. Many candidates seemed to assume that the lack of any information about the direction of the force at *B* was an omission rather than a hint on how to proceed.

Those candidates who started by taking moments about *B* usually reached the required answer without difficulty. The most common errors involved confusion between sine and cosine, and inappropriate accuracy in the final answer after using a decimal approximation for *g*.

Alternative methods involving the force at *B* rarely produced a complete solution. Many candidates assumed that the direction of this force involved the angle α, thus simplifying the algebraic manipulation of their force and moment equations. Those who introduced an unknown angle usually struggled to reach a valid answer, although a handful of concise, correct solutions were seen.

Question 7

Very few candidates noticed the link between part (a) of this questions and parts (b) and (c). This resulted in a considerable quantity of valid but unnecessary work. The marks allocated to the three parts of the question should give candidates an indication that each of the later parts is not expected to involve as much work as the first part.

Part (a) There were many substantially correct answers to this part. Most candidates formed correct equations using restitution and conservation of momentum. The difficulties started with the speed of P – many candidates whose answer was the negative of the printed answer did not justify the change of sign using the information about the value of e, and those whose answer agreed did not appreciate the need to verify that their value for velocity was in fact positive.

Candidates who changed the sign of their answer for the speed of P often went on to substitute incorrectly to find the speed of Q. There was also evidence of some confusion over the exact meaning of the question in part (aii), with several candidates starting by substituting in place of e.

Part (b) Most candidates elected to start the question afresh rather than use the results from part (a). Examiners were presented with confusing diagrams which were often contradicted by the working which followed. Alternatively there was no diagram and we had to decide for ourselves which direction the candidate assumed sphere B would move in after the collision with C.

Part (c) By this stage in the working many candidates were working with an incorrect initial speed of B, and were then further confused about the possible directions of motion of A and B after their collision. This often resulted in a page or more of working to deduce a velocity for B after the collision with A, all for a potential score of one mark. Most candidates did demonstrate an understanding that they needed to compare the speeds of B and C to determine whether or not there would be a further collision.

Question 8

Completely correct solutions to this question were rare, with parts (b) and (c) proving to be a better source of marks than parts (a) or (d).

Part (a) There are several possible methods for finding the maximum speed in this interval. The expected method was to differentiate, find the value of t for which the acceleration is equal to zero, and use this to find the corresponding value of *v*. Candidates using this approach sometimes got as far as the value for *t* and then stopped as if they thought they had answered the question. As an alternative, candidates who recognised this as part of a parabola, either went on to complete the square (with considerable success despite the nature of the algebra involved), or found the average of the two times when the speed is zero to locate the time for maximum speed and hence the speed, or simply quoted formulae for the location of the turning point. Many candidates simply substituted integer values of *t* in to the formula for *v* and stated their largest answer. This alone was not sufficient. Although it is possible to arrive at the correct answer using trial and improvement, most candidates who embarked on this route failed to demonstrate that their answer was indeed a maximum – they usually offered a sequence of increasing values, but did not demonstrate that they had located the turning point in an interval of appropriate width.

Part (b) Many candidates answered this correctly – even those who did not differentiate in part (a) did choose to integrate here. There is a false method, assuming constant speed throughout the interval, which gives the answer 32 incorrectly by finding the speed when $t = 4$ and multiplying the result by 4 – many candidates used this without considering the possibility of variable speed and acceleration.

Part (c) This was usually answered correctly, but some candidates appeared to think that they were being asked to find out when $8t - \frac{3}{2}t^2 = 0$ or when $8\text{t} - \frac{3}{2}t^2 = 16 - 2t$

Part (d) Those candidates who realised that the particle was now moving with uniform acceleration had the simple task of finding the area of two triangles, assuming that they appreciated the significance of $v < 0$ for $t > 8$. Alternatively they could use the equations for motion under uniform acceleration, with the same proviso. For the great majority of candidates, this was about integration and choosing appropriate limits. The integration itself was usually correct, but common errors included ignoring the lower limit of the interval, or not using $s = 32$ when $t = 4$, and stopping after using the upper limit of $t = 10$. Some candidates thought that the limits for t should be from $t = 0$ to $t = 6$, and a large number thought that they should be starting from $t = 5$. Very few of the candidates who found the integral went on to consider what happened between $t = 8$ and $t = 10$.

Statistics for M2 Practice Paper Gold 3

Qu	Max Score	Modal score	Mean %	ALL	A*	A	B	C	D	E	U
				Mean average scored by candidates achieving grade:							
1	5		76.0	3.80		4.30	3.73	3.40	3.08	2.69	1.88
2	7	7	68.7	4.81	6.16	5.61	4.75	3.82	3.06	2.50	1.56
3	8		52.0	4.16	5.70	4.92	3.85	3.06	2.43	1.82	1.15
4	11		42.3	4.65	7.67	5.04	3.45	2.89	1.89	2.16	1.32
5	8		42.1	3.37		3.80	1.32	1.19	0.83	0.76	0.30
6	7		49.3	3.45		3.67	2.37	1.85	1.57	1.36	0.90
7	13		53.4	6.94		9.24	6.98	5.31	3.79	2.64	1.36
8	16		54.8	8.76		11.47	8.61	7.17	5.92	4.69	2.67
	75		**53.2**	**39.94**		**48.05**	**35.06**	**28.69**	**22.57**	**18.62**	**11.14**