

Take $g = 9.8 \text{ ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. A car accelerates from 0 to 108 km h^{-1} in 7.5 seconds. Find its acceleration in ms^{-2} . (3 marks)

2. A book rests on a rough desk-lid which is hinged at one end. When the lid is raised so that it makes an angle of 15° with the horizontal, the book is just on the point of sliding down a line of greatest slope. Modelling the book as a particle, find
 - (a) the coefficient of friction between the book and the desk-lid, (2 marks)
 - (b) the acceleration with which the book starts to move if it is released from rest when the lid is inclined at 20° to the horizontal. (4 marks)

3. A particle P is projected vertically upwards from ground level at time $t = 0$ with speed 20 ms^{-1} . Two seconds later another particle Q is projected vertically upwards with speed 30 ms^{-1} from a point on the same horizontal ground.
 - (a) Taking the upward direction as positive, write down expressions in terms of g and t for the velocities of P and of Q at time t seconds after P is projected. (3 marks)
 - (b) Find the value of t when both particles are moving with the same speed. (4 marks)

4. A jet of water issues from a cylindrical pipe with a circular cross-section of radius 2.75 cm . The water strikes a vertical wall at a speed of 9 ms^{-1} . Taking the density of water to be 1000 kg m^{-3} , calculate
 - (a) the momentum destroyed each second by the impact with the wall, (5 marks)
 - (b) the magnitude of the force exerted by the water on the wall. (2 marks)
 - (c) State one modelling assumption that you have made. (1 mark)

5. Two particles A and B , of mass 1 kg and $m \text{ kg}$ respectively, where $m > 1$, are attached to the ends of a light inextensible string which passes over a small fixed smooth pulley. The particles are released from rest and move with the string taut and vertical.
 - (a) Show that the acceleration of the system is equal to $\frac{(m-1)g}{m+1}$. (6 marks)
 - (b) Find the tension in the string, in terms of m and g , expressing your answer as a single algebraic fraction in its simplest form. (2 marks)When the system is released from rest, both particles are 52.5 cm above ground level and 60 cm below the level of the pulley. B hits the ground after half a second.
 - (c) Find the value of m . (5 marks)
 - (d) Find the speed with which B hits the ground. (3 marks)

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6. At noon, two boats P and Q have position vectors $(\mathbf{i} + 7\mathbf{j})$ km and $(3\mathbf{i} - 8\mathbf{j})$ km respectively relative to an origin O , where \mathbf{i} and \mathbf{j} are unit vectors in the directions due East and due North respectively. P is moving with constant velocity $(3\mathbf{i} - 4\mathbf{j})$ km h⁻¹ and Q is moving with constant velocity $(6\mathbf{i} + 5\mathbf{j})$ km h⁻¹.

(a) Find the position vector of each boat at time t hours after noon, giving your answers in the form $f(t)\mathbf{i} + g(t)\mathbf{j}$, where $f(t)$ and $g(t)$ are linear functions of t to be found. (6 marks)

(b) Find, in terms of t , the distance between the boats t hours after noon. (5 marks)

(c) Calculate the time when the boats are closest together and find the distance between them at this time. (6 marks)

7. A particle starts from rest and accelerates at a uniform rate over a distance of 12 m. It then travels at a constant speed of u ms⁻¹ for a further 30 seconds. Finally it decelerates uniformly to rest at 1.6 ms⁻².

(a) Sketch the velocity-time graph for this motion. (2 marks)

(b) Show that the total time for which the particle is in motion is

$$\frac{5u}{8} + 30 + \frac{24}{u} \text{ seconds.} \quad (5 \text{ marks})$$

(c) Find, in terms of u , the total distance travelled by the particle during the motion. (3 marks)

(d) Given that the total time for the motion is 39.5 seconds, show that $5u^2 - 76u + 192 = 0$. (3 marks)

(e) Find the two possible values of u and the total distance travelled in each case. (5 marks)