GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P6

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.
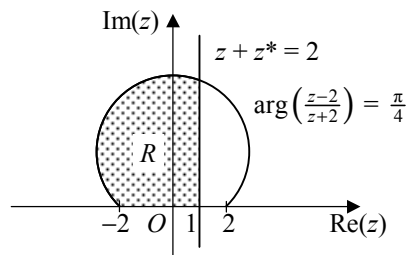
P6 Paper H – Marking Guide

1.   assume true for $n = k$ $\therefore t_k = 7 - 4k$
     $\therefore t_{k+1} = t_k - 4 = 7 - 4k - 4 = 7 - 4(k + 1)$ M1 A2
     $\therefore$ true for $n = k + 1$ if true for $n = k$
     if $n = 1$ $7 - 4n = 7 - 4 = 3$ as required $\therefore$ true for $n = 1$ B1
     $\therefore$ by induction true for $n \in \mathbb{Z}^+$ A1   **(5)**

2.   *(a)* (i), (ii); *(b)*

     $z + z^* = 2$; $x + iy + x - iy = 2$; $2x = 2$ $\therefore x = 1$ M1 A1

     $z + z^* = 2$ B1

     $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$ B3

                                                                        M1 A1   **(8)**

3.   *(a)*

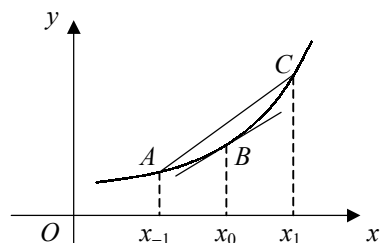   B1

     grad. of tangent at $B \approx$ grad. of chord AC M1
     $\therefore f'(x_0) \approx \dfrac{y_1 - y_{-1}}{x_1 - x_{-1}} = \dfrac{f(x_0 + h) - f(x_0 - h)}{2h}$ A1

     (b) $f'(x_0) = \sqrt{2x_0 + f(x_0)} \approx \dfrac{f(x_1) - f(x_{-1})}{0.4}$ M1

     $f(x_1) = f(x_{-1}) + 0.4\sqrt{2x_0 + f(x_0)}$ A1
     $x_{-1} = 0$, $x_0 = 0.2$, $x_1 = 0.4$; $f(x_{-1}) = 1$, $f(x_0) = 1.25$, $f(x_1) = ?$
     $\therefore f(x_1) = 1 + 0.4\sqrt{(0.4 + 1.25)} = 1.5138...$ A1
     $f(x_2) = f(x_0) + 0.4\sqrt{2x_1 + f(x_1)}$ M1
     $x_0 = 0.2$, $x_1 = 0.4$, $x_2 = 0.6$; $f(x_0) = 1.25$, $f(x_1) = 1.5138...$, $f(x_2) = ?$
     $\therefore f(x_2) = 1.25 + 0.4\sqrt{(0.8 + 1.5138...)} = 1.8584...$ $\therefore f(0.6) \approx 1.86$ (3sf)   A1   **(8)**

© Solomon Press

4. (a) $\vec{AB} = -\mathbf{i} + (q+1)\mathbf{j} - 4\mathbf{k}, \ \vec{AC} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ M1 A1

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & q+1 & -4 \\ 1 & -3 & 4 \end{vmatrix}$$

$= \mathbf{i}(4q + 4 - 12) - \mathbf{j}(-4 + 4) + \mathbf{k}(3 - q - 1) = (4q - 8)\mathbf{i} + (2 - q)\mathbf{k}$ M1 A2

(b) $\vec{AB} \times \vec{AC} = 4(q - 2)\mathbf{i} + (2 - q)\mathbf{k} = (q - 2)[4\mathbf{i} - \mathbf{k}]$ M1

$\therefore \ \mathbf{n} = 4\mathbf{i} - \mathbf{k}$ is perp. to Π A1

(c) $\mathbf{r}.(4\mathbf{i} - \mathbf{k}) = (2\mathbf{i} - \mathbf{j} + \mathbf{k}).(4\mathbf{i} - \mathbf{k}) = 8 - 1 = 7$ M1

$\therefore$ eqn. is $\mathbf{r}.(4\mathbf{i} - \mathbf{k}) = 7$ A1

(d) $q = -1, \ \vec{AB} \times \vec{AC} = -12\mathbf{i} + 3\mathbf{k}$

volume $= \frac{1}{6} \left| \vec{AB} \times \vec{AC} . \vec{AO} \right|$ M1

$\qquad = \frac{1}{6} \left| (-12\mathbf{i} + 3\mathbf{k}).(-2\mathbf{i} + \mathbf{j} - \mathbf{k}) \right|$

$\qquad = \frac{1}{6} \left| 24 - 3 \right| = \frac{1}{6} \times 21 = 3.5 \ \text{units}^3$ M1 A1   **(12)**

---

5. (a) $(\cos\theta + \mathrm{i}\sin\theta)^5 = \cos 5\theta + \mathrm{i}\sin 5\theta$

$\quad = \cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta + 10\mathrm{i}^2\cos^3\theta\sin^2\theta + 10\mathrm{i}^3\cos^2\theta\sin^3\theta$

$\qquad\qquad\qquad + 5\mathrm{i}^4\cos\theta\sin^4\theta + \mathrm{i}^5\sin^5\theta$ M1 A1

equating real parts:

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ M1 A1

$\qquad = \cos^5\theta - 10\cos^3\theta(1 - \cos^2\theta) + 5\cos\theta(1 - \cos^2\theta)^2$ M1

$\qquad = \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$

$\qquad = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

$\qquad = \cos\theta(16\cos^4\theta - 20\cos^2\theta + 5)$ A1

(b) $\cos 5\theta = 0, \ 5\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ M1

$\qquad \therefore \ \theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \ldots$ A1

if $\cos\theta(16\cos^4\theta - 20\cos^2\theta + 5) = 0$

then $\cos\theta = 0$ giving $\theta = \frac{\pi}{2}, \ldots$ or $16\cos^4\theta - 20\cos^2\theta + 5 = 0$ M1

using quad formula, $\cos^2\theta = \dfrac{20 \pm \sqrt{80}}{32} = \dfrac{20 \pm 4\sqrt{5}}{32} = \dfrac{5 \pm \sqrt{5}}{8}$ M1 A1

$\cos^2(\frac{\pi}{10}) > \cos^2(\frac{3\pi}{10}) \ \therefore \ \cos^2(\frac{3\pi}{10}) = \dfrac{5 - \sqrt{5}}{8}$ M1 A1   **(13)**

---

6. *(a)*   let $f(x) \equiv \ln\left(\frac{1+x}{1-2x}\right) = \ln(1+x) - \ln(1-2x)$ M1

$f'(x) = \frac{1}{1+x} - \frac{1}{1-2x}(-2) = \frac{1}{1+x} + \frac{2}{1-2x}$ M1 A1

$f''(x) = -(1+x)^{-2} - 2(1-2x)^{-2}(-2) = \frac{4}{(1-2x)^2} - \frac{1}{(1+x)^2}$ M1 A1

$f'''(x) = -8(1-2x)^{-3}(-2) + 2(1+x)^{-3} = \frac{2}{(1+x)^3} + \frac{16}{(1-2x)^3}$ A1

(b) $f(0) = 0,\ f'(0) = 1 + 2 = 3,\ f''(0) = 4 - 1 = 3,\ f'''(0) = 2 + 16 = 18$ M1 A1

$\therefore \ \ln\left(\frac{1+x}{1-2x}\right) = 0 + 3x + 3(\frac{1}{2!})x^2 + 18(\frac{1}{3!})x^3 + \dots$ M1

$= 3x + \frac{3}{2}x^2 + 3x^3 + \dots$ A1

(c) $-1 < x \le 1$ and $-1 \le 2x < 1 \ \therefore \ -\frac{1}{2} \le x < \frac{1}{2}$ B1

(d) $\frac{1+x}{1-2x} = \frac{4}{3} \ \Rightarrow \ 3 + 3x = 4 - 8x$ giving $x = \frac{1}{11}$ M1 A1

$\therefore \ \ln\frac{4}{3} \approx \frac{3}{11} + \frac{3}{2} \times \frac{1}{121} + 3 \times \frac{1}{1331} = 0.287$ (3dp)   A1   **(14)**

---

7. *(a)* $(\mathbf{B} - 2\mathbf{I})\mathbf{A} = \begin{pmatrix} 4 & 5 & 2 \\ -1 & -2 & -2 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & a & 2 \\ -1 & b & -2 \\ 0 & 0 & c \end{pmatrix} = 3\mathbf{I} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ M1

$\therefore \ 4a + 5b + 0 = 0$
$-a - 2b + 0 = 3$ M1 A1
solve. simul. giving $a = 5, b = -4$ M1 A1
$0 + 0 + 3c = 3, \ c = 1$ A1

(b) $\frac{1}{3}(\mathbf{B} - 2\mathbf{I})\mathbf{A} = \mathbf{I} \ \therefore \ \mathbf{A}^{-1} = \frac{1}{3}(\mathbf{B} - 2\mathbf{I})$ M1

$= \frac{1}{3}\begin{pmatrix} 4 & 5 & 2 \\ -1 & -2 & -2 \\ 0 & 0 & 3 \end{pmatrix}$ A1

(c) $\begin{pmatrix} 2 & 5 & 2 \\ -1 & -4 & -2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ M1

$\therefore \ 2x + 5y + 2z = x$
$-x - 4y - 2z = y$
$z = z$ A1
giving $x + 5y + 2z = 0$ M1 A1

(d) $\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 4 & 5 & 2 \\ -1 & -2 & -2 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} -3 \\ 0 \\ 9 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$ M1 A1

$p = -1, q = 0, r = 3$ A1   **(15)**

Total   **(75)**

© *Solomon Press*

Performance Record – P6 Paper H

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	proof by induction	complex loci	step-by-step soln. of diff. eqn.	vectors	complex nos., De Moivre's	Maclaurin series	matrices, inverse	
Marks	5	8	8	12	13	14	15	75
Student								