GCE Examinations

Advanced Subsidiary / Advanced Level

# Pure Mathematics
# Module P6

Paper B

# MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

*Written by Rosemary Smith & Shaun Armstrong*

# P6 Paper B – Marking Guide

**1.** $\ln(1 + ax) \times (1 + bx)^{-1} = (ax - \frac{1}{2}a^2x^2 + \ldots)(1 - bx + \ldots)$ B1

$= ax - abx^2 - \frac{1}{2}a^2x^2 + \ldots$ M1 A1

$\therefore\ ax + (-ab - \frac{1}{2}a^2)x^2 = 3x + \frac{3}{2}x^2$

$\therefore\ a = 3$ and $-ab - \frac{1}{2}a^2 = \frac{3}{2}$ M1

giving $-3b - \frac{9}{2} = \frac{3}{2}$ so $a = 3,\ b = -2$ A1 **(5)**

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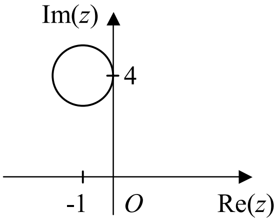
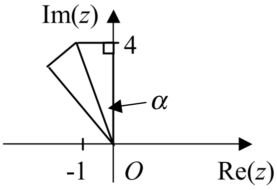
**2.** *(a)*




circle centre $-1 + 4i$, radius 1 B2

*(b)*




$\tan\alpha = \frac{1}{4},\ \alpha = 14.036\ldots°$ M1

max. $\arg z = 90° + 2\alpha = 118.1°$ (1dp) M1 A1 **(5)**

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**3.** *(a)* $\cosh ix = \frac{1}{2}(e^{ix} + e^{-ix}) = \frac{1}{2}[\cos x + i\sin x + \cos(-x) + i\sin(-x)]$ M1

$= \frac{1}{2}[\cos x + i\sin x + \cos x - i\sin x] = \cos x$ A1

*(b)* $\cosh ix = e^{ix}$ $\therefore\ \cos x = \cos x + i\sin x$ M1

$\therefore\ \sin x = 0$ giving $x = 0, \pi$ M1 A1 **(5)**

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**4.** assume true for $n = k$ and $n = k + 1$ $\therefore\ u_k = 2^k,\ u_{k+1} = 2^{k+1}$ M1

$\therefore\ u_{k+2} = 5(2^{k+1}) - 6(2^k)$ M1

$= 10(2^k) - 6(2^k) = 4(2^k) = 2^{k+2}$ M1 A1

$\therefore$ true for $n = k + 2$ if true for $n = k$ and $n = k + 1$

if $n = 1,\ u_1 = 2^1 = 2$; if $n = 2,\ u_2 = 2^2 = 4$ $\therefore$ true for $n = 1$ and $n = 2$ B1

$\therefore$ by induction true for integer $n,\ n \geq 1$ A1 **(6)**

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**5.** *(a)* $\lambda = -1,\ \begin{vmatrix} 2 & 2 & -1 \\ 0 & 2 & -4 \\ x & 3 & 0 \end{vmatrix} = 0$ M1

$\therefore\ 2(0 + 12) - 2(0 + 4x) - 1(0 - 2x) = 0$ M1

$24 - 8x + 2x = 0$ so $x = 4$ A1

*(b)* $\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & -4 \\ 4 & 3 & -1-\lambda \end{vmatrix} = 0$ M1

$(1 - \lambda)[(1 - \lambda)(-1 - \lambda) + 12] - 2(0 + 16) - 1[0 - 4(1 - \lambda)] = 0$ A1

$-(1 + \lambda)(1 - \lambda)^2 + 12 - 12\lambda - 32 + 4 - 4\lambda = 0$ M1

$-(1 + \lambda)(1 - \lambda)^2 - 16\lambda - 16 = 0$

$(1 + \lambda)(1 - \lambda)^2 + 16(1 + \lambda) = 0$

$(1 + \lambda)[(1 - \lambda)^2 + 16] = 0$ A1

$\lambda = -1$; or $(1 - \lambda)^2 = -16$, not poss. for real $\lambda$ M1

$\therefore\ \lambda = -1$ is only real eigenvalue A1

*(c)* $\begin{pmatrix} 2 & 2 & -1 \\ 0 & 2 & -4 \\ 4 & 3 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$2x + 2y - z = 0$ $\quad \therefore 2x + 4z - z = 0$

$2y - 4z = 0$ $\quad 2x + 3z = 0$ $\quad \therefore$ eigenvector $k\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ M1 A1 **(11)**

$\therefore y = 2z$ $\quad \therefore 2x = -3z$

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**6.** *(a)* $\dfrac{y_1 - y_0}{0.1} \approx x_0 y_0 \quad \therefore\ y_1 \approx 0.1x_0y_0 + y_0$ M1 A1

$x_0 = 0.2,\ x_1 = 0.3,\ y_0 = 1 \quad \therefore\ y_1 \approx 1.02$ M1 A1

$y_2 \approx 0.1x_1y_1 + y_1$ M1

$x_1 = 0.3,\ x_2 = 0.4,\ y_1 = 1.02 \quad \therefore\ y_2 \approx 1.0506$ A1

*(b)* $\displaystyle\int_1^y \frac{1}{y}\, dy = \int_{0.2}^{0.4} x\, dx$ M1

$[\ \ln|y|\ ]_1^y = [\ \tfrac{1}{2}x^2\ ]_{0.2}^{0.4}$ A1

$\ln|y| - \ln 1 = 0.08 - 0.02$ giving $y = e^{0.06}$ M1 A1

*(c)* % error $= \dfrac{e^{0.06} - 1.0506}{e^{0.06}} \times 100\% = 1.1\%$ (1dp) M1 A1 **(12)**

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| **7.** | *(a)* | $z^n + \frac{1}{z^n} = \cos n\theta + \text{i}\sin n\theta + \cos(-n\theta) + \text{i}\sin(-n\theta)$ | M1 | |
| | | $= \cos n\theta + \text{i}\sin n\theta + \cos n\theta - \text{i}\sin n\theta = 2\cos n\theta$ | A1 | |
| | | $z^n - \frac{1}{z^n} = \cos n\theta + \text{i}\sin n\theta - \cos n\theta + \text{i}\sin n\theta = 2\text{i}\sin n\theta$ | A1 | |
| | *(b)* | $(z + \frac{1}{z})^4 = z^4 + 4z^3\frac{1}{z} + 6z^2\frac{1}{z^2} + 4z\frac{1}{z^3} + \frac{1}{z^4}$ | M1 | |
| | | $(2\cos\theta)^4 = 2\cos 4\theta + 4(2\cos 2\theta) + 6$ | M1 A1 | |
| | | $16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$ | A1 | |
| | | $(z - \frac{1}{z})^4 = z^4 - 4z^2 + 6 - \frac{4}{z^2} + \frac{1}{z^4}$ | M1 | |
| | | $(2\text{i}\sin\theta)^4 = 2\cos 4\theta - 8\cos 2\theta + 6$ | | |
| | | $16\sin^4\theta = 2\cos 4\theta - 8\cos 2\theta + 6$ | A1 | |
| | | $\therefore\ 16(\cos^4\theta + \sin^4\theta) = 4\cos 4\theta + 12$ | M1 | |
| | | $\cos^4\theta + \sin^4\theta = \frac{1}{4}\cos 4\theta + \frac{3}{4}$ so $A = \frac{1}{4},\ B = \frac{3}{4}$ | A1 | |
| | *(c)* | $\text{I} = \int_0^{\frac{\pi}{8}} \frac{1}{4}\cos 4\theta + \frac{3}{4}\ d\theta$ | | |
| | | $= [\frac{1}{16}\sin 4\theta + \frac{3}{4}\theta]_0^{\frac{\pi}{8}}$ | M1 A1 | |
| | | $= \frac{1}{16} + \frac{3}{32}\pi$ | A1 | **(14)** |
| **8.** | *(a)* | $\overrightarrow{AB} = -5\mathbf{i} + \mathbf{j} - 3\mathbf{k},\ \overrightarrow{AC} = -2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ | M1 A1 | |
| | | $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 1 & -3 \\ -2 & 3 & 4 \end{vmatrix}$ | | |
| | | $= \mathbf{i}(4 + 9) - \mathbf{j}(-20 - 6) + \mathbf{k}(-15 + 2) = 13\mathbf{i} + 26\mathbf{j} - 13\mathbf{k}$ | M1 A2 | |
| | *(b)* | $\overrightarrow{AD} = -4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ | B1 | |
| | | volume $= \frac{1}{6}\lvert(13\mathbf{i} + 26\mathbf{j} - 13\mathbf{k}).(-4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})\rvert$ | M1 | |
| | | $= \frac{13}{6}\lvert -4 - 8 - 6\rvert = 39$ units$^3$ | A1 | |
| | *(c)* | $\mathbf{n} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ | B1 | |
| | | $\mathbf{r.n} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}).(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 3 - 2 - 2 = -1$ | M1 | |
| | | $\therefore$ eqn. is $\mathbf{r}.(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = -1$ | A1 | |
| | *(d)* | line through *DE* has eqn. $\mathbf{r} = -\mathbf{i} - 5\mathbf{j} + 8\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ | M1 A1 | |
| | | at intersection $[(-1 + \lambda)\mathbf{i} + (-5 + 2\lambda)\mathbf{j} + (8 - \lambda)\mathbf{k}].(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = -1$ | M1 | |
| | | $-1 + \lambda - 10 + 4\lambda - 8 + \lambda = -1$ giving $\lambda = 3$ | M1 A1 | |
| | | $\therefore$ *E* is (2, 1, 5) | A1 | **(17)** |
| | | | Total | **(75)** |

# Performance Record – P6 Paper B

| Question no. | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | Total |
|---|---|---|---|---|---|---|---|---|---|
| Topic(s) | Maclaurin series | complex loci | complex nos. | proof by induction | matrices, eigenvals. | step-by-step soln. of diff. eqn. | complex nos., De Moivre's | vectors | |
| Marks | 5 | 5 | 5 | 6 | 11 | 12 | 14 | 17 | 75 |
| Student | | | | | | | | | |
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