PURE MATHS 6 (A) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1.	Line from (0, 2) sloping upward at 60°	B3	3
2.	$\left(\frac{d^2y}{dx^2}\right)_0 = -1{\cdot}5 \approx \frac{y_1 - 0 - 1}{0.5^2}$ $\quad y_1 \approx -0{\cdot}375 + 1 = 0{\cdot}625$	M1 A1 M1 A1 A1	5
3.	$0 = 6 \times 0$, so true for $n = 1$ Assume true for $n = k$, so $k^3 - k = 6m$	B1 M1	
	Then $(k+1)^3 - (k+1) = k^3 + 3k^2 + 2k = k^3 - k + 3(k^2 + k)$	M1 A1	
	$= 6m + 3k(k+1)$, which is a multiple of 6 as $k(k+1)$ is even	A1 A1	6
4.	(a) $\ln(2+x) - \ln(1-x) = \ln 2 + \ln(1 + x/2) - \ln(1-x)$	M1 M1	
	$= \ln 2 + (x/2 - x^2/8 + x^3/24) - (-x - x^2/2 - x^3/3)$	A1 A1	
	$= \ln 2 + 3x/2 + 3x^2/8 + 3x^3/8$	A1	
	(b) $x = 1/2$: $\ln 5 = \ln 2 + 3/4 + 3/32 + 3/64 + \ldots = \ln 2 + 57/64 + \ldots$	M1 A1	
	$\ln 5 - \ln 2 > 57/64$ $\quad \ln(5/2) > 57/64$	M1 A1	9
5.	(a) $f''(0) = -1$ $\quad y''' = yy'' + y'^2$ $\quad f'''(0) = -1 + 1 = 0$	B1 B1	
	$y^{iv} = yy''' + y'y'' + 2y'y''$ $\quad f^{iv}(0) = 1 + 2 = 3$	M1 A1	
	$y^{v} = yy^{iv} + y'''y' + 3(y'y''' + y''^2)$ $\quad f^{v}(0) = 3 + 3 = 6$	M1 A1 A1	
	$y = 1 - x - x^2/2 + x^4/8 + x^5/20$	M1 A1	
	(b) $y(0{\cdot}1) \approx 0{\cdot}895$	M1 A1	11
6.	(a) $\begin{vmatrix} -\lambda & 4 & 3 \\ -6 & 1-\lambda & 6 \\ 2 & 4 & 1-\lambda \end{vmatrix} = 0$ $\quad \begin{vmatrix} -\lambda & 4 & 3-\lambda \\ -6 & 1-\lambda & 0 \\ 2+\lambda & 0 & 0 \end{vmatrix} = 0$	M1 M1 A1	
	$(3-\lambda)(0 - (1-\lambda)(2+\lambda)) = 0$ $\quad \lambda = -2, 1, 3$	A1 A1	
	(b) $4y + 3z = x$, $\quad -6x + y + 6z = y$, $\quad 2x + 4y + z = z$	M1 A1	
	Hence $x = -2y$, $z = -2y$ $\quad$ Eigenvector $(-2 \quad 1 \quad -2)$	M1 A1	
	(c) $\mathbf{D} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ or equivalent	B1	
	Columns of **P** are eigenvectors corresponding to −2, 1, 3	B1	11
7.	(a) $-2\mathbf{i} - 14\mathbf{j} + 6\mathbf{k}$, $\quad -4\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}$	M1 A1 A1	
	(b) $2 - 28 + 12 = -16 + 7 - 5$ $\quad -14 = -14$ ✓	M1 A1	
	(c) Volume $= 1/6 \times \lvert -14 \rvert = 7/3$	M1 A1	
	(d) Plane ABC is $\mathbf{r} = (4 \quad -1 \quad -1) + s(1 \quad -1 \quad -2) + t(3 \quad -1 \quad 1)$	M1 A1	
	$4 + s + 3t = x$, $\quad -1 - s - t = y$, $\quad -1 - 2s + t = z$	A1 A1	
	$s = (1 - x - 3y)/2 = (-2 - y - z)/3$ $\quad 3x + 7y - 2z = 7$	M1 A1	
	In vector form: $\mathbf{r} \cdot (3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) = 7$	A1	14
8.	(a) $u + iv = (x + iy - 1)/(x + iy) = (x^2 + y^2 - x + iy)/(x^2 + y^2)$	M1 A1 A1	
	$\lvert z \rvert^2 = x^2 + y^2 = 1$, so $u + iv = 1 - x + iy$	M1 A1	
	$x = 1 - u$, $y = v$ $\quad (1-u)^2 + v^2 = 1$ $\quad u^2 + v^2 - 2u = 0$	M1 A1	
	(b) Centre (1, 0), radius 1, so $\lvert w - 1 \rvert = 1$	M1 A1	
	(c) If $\arg w = \pi/4$, $u = v$ and $u, v > 0$	B1 B1	
	$w = 1 - 1/z$ so $u + iv = (1 - x/[x^2 + y^2]) + iy/[x^2 + y^2]$	M1 A1	
	so $1 - x/(x^2 + y^2) = y/(x^2 + y^2)$, i.e. $x^2 + y^2 = x + y$	M1 A1	
	$x^2 + y^2 - x - y = 0$ $\quad$ Circle centre (1/2, 1/2), radius $1/\sqrt{2}$	A1	16