1. The complex number $z$ is such that $|z-\mathrm{i}| = |z+2\mathrm{i}|$.
   (a) Sketch the locus of $z$ in an Argand diagram. **(2 marks)**
   (b) State the value of the imaginary part of $z$. **(1 mark)**
   (c) Find the value of the real number $k$ for which the above locus and the locus $|z-2| = k$ intersect at only one point. **(2 marks)**

2. (a) Write down the Maclaurin series for $e^{-x}$ as far as the term in $x^4$. **(2 marks)**
   (b) Show that if $x$ is small enough for terms in $x^3$ to be negligible, then

$$e^{-x}(\sin x + \cos x) \approx 1 - x^2.$$

**(5 marks)**

3. (a) Show that the matrix

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

   has exactly two non-zero eigenvalues. **(6 marks)**
   (b) State the significance of the fact that zero is an eigenvalue of M. **(1 mark)**

4. $O$ is the origin and $A$, $B$ and $C$ are the points with position vectors $\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ respectively relative to $O$.
   (a) Find $\overrightarrow{OA} \times \overrightarrow{OB}$. **(3 marks)**
   (b) Find the volume of the parallelepiped which has $OA$, $OB$ and $OC$ as three of its edges. **(4 marks)**

5. $\dfrac{d^2y}{dx^2} + 3xy = 0$. When $x = 0$, $y = 1$ and $\dfrac{dy}{dx} = -1$.
   (a) Obtain the Taylor's series expansion of $y$ in ascending powers of $x$ as far as the term in $x^4$. **(7 marks)**
   (b) Hence estimate, to 4 decimal places, the value of $y$ when $x = 0{\cdot}1$. **(3 marks)**

6. $S$ and $T$ are two linear transformations of the $x$-$y$ plane. $S$ is a positive (anti-clockwise) quarter-turn about the origin (0, 0) and $T$ is the transformation represented by the matrix

$$\mathbf{N} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}.$$

(a) Write down the matrix **M** of the transformation $S$. **(2 marks)**

(b) Write down the eigenvalues of **N**. **(2 marks)**

(c) Describe the effect of the transformation $T$ on a general point $(x, y)$. **(2 marks)**

(d) Find the matrix which represents the composite transformation '$S$ followed by $T$'. **(2 marks)**

(e) If $\mathbf{M}^{-1}$ and $\mathbf{N}^{-1}$ were given, describe how you could use them to find $(\mathbf{MN})^{-1}$. **(2 marks)**

7. (a) Find, in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$, an equation of the plane $\Pi$ containing the points $P(-2, 1, 3)$, $Q(1, 1, -1)$ and $R(2, 4, -2)$. **(3 marks)**

(b) Express the equation of $\Pi$ in the cartesian form $ax + by + cz = d$. **(5 marks)**

(c) Find, in the form $(\mathbf{r} - \mathbf{u}) \times \mathbf{v} = \mathbf{0}$, the equation of the line through $P$ perpendicular to $\Pi$. **(3 marks)**

(d) Find the shortest distance from $P$ to a plane parallel to $\Pi$ which contains the origin. **(3 marks)**

8. (a) Use de Moivre's theorem to prove that $\sin 6\theta = \sin\theta \cos\theta\,(32\cos^4\theta - 32\cos^2\theta + 6)$. **(7 marks)**

(b) Express $\dfrac{\sin 6\theta}{\sin 2\theta}$ as a polynomial in $\cos\theta$, for $\sin 2\theta \neq 0$. **(3 marks)**

(c) Hence or otherwise solve the equation $\sin 2\theta + \sin 6\theta = 0$, for $0 < \theta < \pi$, giving the solutions in terms of $\pi$. **(5 marks)**