PURE MATHS 6 (A) TEST PAPER 4 : ANSWERS AND MARK SCHEME

1.	$f(0) = 1$ $\quad f'(x) = \sec^2(x + \pi/4)$ $\quad f'(0) = 2$	M1 A1	
	$f''(x) = 2\sec^2(x + \pi/4)\tan(x + \pi/4)$ $\quad f''(0) = 4$	M1 A1	
	Series is $1 + 2x + 2x^2$	A1	5
2.	$r^4 e^{4i\theta} = 16i$ $\quad r = 2,\ 4\theta = \pi/2, 5\pi/2, -3\pi/2, -7\pi/2$	M1 A1	
	Roots are $2e^{-7i\pi/8},\ 2e^{-3i\pi/8},\ 2e^{i\pi/8},\ 2e^{5i\pi/8}$	A1 A1 A1 A1	6
3.	(a) $\mathbf{a} \times \mathbf{b} = -8\mathbf{i} - \mathbf{j} + 5\mathbf{k}$	M1 A1	
	(b) Area $= 1/2\ \lvert \mathbf{a} \times \mathbf{b} \rvert = [\sqrt{(64 + 1 + 25)}]/2 = (3\sqrt{10})/2$	M1 A1	
	(c) $(\mathbf{r} - \mathbf{u}) \times (\mathbf{b} - \mathbf{a}) = 0$, e.g. $(\mathbf{r} - [2\mathbf{i} - \mathbf{j} + 3\mathbf{k}]) \times (3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) = 0$	M1 A1 A1	7
4.	$\mathbf{M}^1$ has each entry $2^0 x^1$, so true for $n = 1$	M1 A1	
	Assume true for $n = k$ $\quad$ Then $\mathbf{M}^{k+1} = \mathbf{M}.\mathbf{M}^k$ has each entry	M1 M1 A1	
	$= [x(2^{k-1}x^k)] \times 2 = 2^k x^{k+1} = 2^{(k+1)-1}x^{k+1}$, so true for $n = k+1$	M1 A1 A1	8
5.	$y''(0) = 4$ $\quad y''' + xy'' + y' - 3y' = 0$ $\quad y'''(0) = 2$	B1 M1 A1 A1	
	Hence $y = x + 2x^2 + x^3/3$ $\quad y(0{\cdot}15) \approx 0{\cdot}196$	M1 A1 A1 M1 A1	9
6.	(a) $\mathbf{r} \cdot (6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}) = 17$	M1 A1	
	(b) $\lvert 6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k} \rvert = \sqrt{77}$, so perp. distance $= \dfrac{17}{\sqrt{77}}$	M1 M1 A1	
	(c) Angle θ between normals $6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$ and $3\mathbf{i} - \mathbf{j} - \mathbf{k}$ is given by	M1	
	$\cos\theta = (6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}) \cdot (3\mathbf{i} - \mathbf{j} - \mathbf{k}) / (\sqrt{77})(\sqrt{11}) = 0{\cdot}653$	M1 M1 A1	
	$\theta = 49°$	A1	10
7.	(a) $\det(\mathbf{M}) = 5(2) + 1(-46) = -36$	M1 A1	
	$M^{-1} = -\dfrac{1}{36}\begin{pmatrix} 2 & -8 & 1 \\ 12 & -12 & 6 \\ -46 & 40 & -5 \end{pmatrix}$	M1 A1 A1	
	(b) Applying M^{-1} to $(2, 4, -8)$ gives $(1, 2, -3)$	M1 A1 A1	
	(c) Char. equation is $(5 - \lambda)(-\lambda - 1)(-\lambda - 2) + (-48 + 2\lambda + 2) = 0$	M1 A1	
	$-\lambda^3 + 2\lambda^2 + 13\lambda + 10 - 46 + 2\lambda = 0$ $\quad \lambda^3 - 2\lambda^2 - 15\lambda + 36 = 0$	A1	
	$(\lambda + 4)(\lambda^2 - 6\lambda + 9) = 0$ $\quad (\lambda + 4)(\lambda - 3)^2 = 0$	M1 A1	
	Other eigenvalue is 3	A1	14
8.	(a) $z^n = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$	B1	
	$z^{-n} = (\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$	B1	
	Adding, $z^n + z^{-n} = 2\cos n\theta$ $\quad$ Subtracting, $z^n - z^{-n} = 2i\sin n\theta$	M1 A1 A1	
	(b) $(z + z^{-1})^4 = z^4 + 4z^3(z^{-1}) + 6z^2(z^{-1})^2 + 4z(z^{-1})^3 + (z^{-1})^4$	M1 A1	
	$= z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$	A1	
	Hence $(2\cos\theta)^4 = 2\cos 4\theta + 8\cos 2\theta + 6$	M1 A1	
	so $\cos^4\theta = \dfrac{1}{8}\cos 4\theta + \dfrac{1}{2}\cos 2\theta + \dfrac{3}{8}$	M1 A1	
	(c) $\cos^4\theta = \dfrac{1}{8}\cos 4\theta + \dfrac{1}{2}(2\cos^2\theta - 1) + \dfrac{3}{8}$	M1 A1 A1	
	$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	A1	16

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