1. Let a be a fixed integer greater than 1. Prove by induction that, for all integers $n \geq 1$, $a^n - 1$ is divisible by $a - 1$. **(5 marks)**

2. $z_1 = 2e^{i\pi/4}$ and $z_2 = \frac{1}{2}e^{i\pi/3}$.

   (a) Write down the modulus and the argument of (i) $z_1 z_2$, (ii) $\frac{z_1}{z_2}$. **(4 marks)**

   (b) Show points representing z_1 and z_2 on an Argand diagram, and draw on your diagram the locus given by the equation $|z - z_1| = |z - z_2|$. **(3 marks)**

3. Obtain the Taylor expansion of $\cos x$ in ascending powers of $(x - \pi)$, as far as the term in $(x - \pi)^4$. **(8 marks)**

4. (a) With the usual notation, derive the result $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$ **(3 marks)**

   (b) Use this step-by-step method, with a step length of 0·1, to find an approximate value of y when $x = 1{\cdot}2$, given that $\frac{dy}{dx} - y = 2x^2$, $y = 1$ when $x = 1$ and $y = 0{\cdot}73$ when $x = 0{\cdot}9$. **(7 marks)**

5. (a) Find the eigenvalues of the matrix $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$. **(7 marks)**

   (b) Hence write down a diagonal matrix **D** such that, for some non-singular matrix **P** (which need not be found),
   $$\mathbf{D} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}.$$ **(2 marks)**

   (c) Find $\mathbf{A}^{-1}$ and write down the eigenvalues of $\mathbf{A}^{-1}$. **(5 marks)**

6. The plane Π with equation $\mathbf{r}.(3\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = p$ contains the points A (2, 1, c) and B (3, c, 2).

   (a) Find the values of c and p. **(3 marks)**

   (b) Find the perpendicular distance from the origin O to the plane Π. **(3 marks)**

   (c) Find the co-ordinates of the point Π which is the reflection of O in Π. **(4 marks)**

   (d) Find $\overrightarrow{OA} \times \overrightarrow{AB}$ and hence or otherwise find the area of triangle OAB. **(5 marks)**

7. (a) Given that $y = \arccos x$, express x in terms of y and hence obtain $\frac{dx}{dy}$ in terms of y. **(2 marks)**

   (b) Deduce that $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$ and find $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ in terms of x. **(7 marks)**

   (c) Hence write down the first three non-zero terms in the Maclaurin series for $\arccos x$. **(5 marks)**

   (d) Use your series to find an estimate of arccos (0·6) correct to 2 decimal places, showing your working clearly. **(2 marks)**