1. Find the first three terms in the Maclaurin expansion of $\ln(4 - \frac{1}{2}x)$. **(4 marks)**

2. (a) Assuming the result $e^{ix} = \cos x + i \sin x$, prove that $\sinh ix = i \sin x$. **(3 marks)**

   (b) Given that x is real, find the general solution of the equation $\sinh ix = e^{ix}$. **(4 marks)**

3. (a) Write down vectors which are normal to each of the planes

   $$\mathbf{r}.(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 2 \quad \text{and} \quad x + 2y - z = 3.$$ **(2 marks)**

   (b) Hence or otherwise find a direction vector of the line of intersection of these two planes. **(5 marks)**

4. Prove by induction that, for $n \geq 1$, $\sum_{r=1}^{n} 2^{1-r} = 2(1 - 2^{-n})$. **(8 marks)**

5. A transformation from the z-plane to the w-plane is defined by $w = \frac{z-1}{z+i}$, where $w = u + iv$.

   Show that the image of the real axis under this transformation is the circle with equation

   $$u^2 + v^2 - u - v = 0.$$ **(8 marks)**

6. Given that $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y^2 = 0$, and that when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{1}{2}$,

   (a) show that $\frac{d^3y}{dx^3} = 0$ when $x = 0$. **(4 marks)**

   (b) Use the Taylor's series method to express y as a series of ascending powers of x as far as the term in x^4. Hence estimate y when $x = 0{\cdot}1$, giving your answer to 4 significant figures. **(6 marks)**

7. (a) Given that $z = \cos\theta + i\sin\theta$, show that for any positive integer n,

   $$z^n - \frac{1}{z^n} = 2i\sin n\theta.$$ **(4 marks)**

   (b) Deduce an expression for $\sin^7\theta$ in terms of sines of multiples of θ. **(8 marks)**

8. A linear transformation T of $\mathbb{R}^3$ is represented by the matrix **M**.

   The images under T of the points $(2, -1, 1)$, $(1, 3, 0)$ and $(1, 4, 0)$ are respectively $(3, 1, 2)$, $(1, 3, 8)$ and $(1, 4, 10)$.

   (a) Find the matrix **M**. **(5 marks)**

   (b) Find cartesian equations of the image under T of the line $x - 1 = y + 1 = z$. **(5 marks)**

   (c) Find the eigenvalues of **M**. **(5 marks)**

   (d) Find a normalised eigenvector corresponding to the largest eigenvalue of **M**. **(4 marks)**