Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1
Silver Level S1

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
71	63	55	47	39	31

1. $$f(x) = 3^x + 3x - 7$$

(*a*) Show that the equation $f(x) = 0$ has a root α between $x = 1$ and $x = 2$.

(2)

(*b*) Starting with the interval [1, 2], use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

June 2011

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2. $$z = 2 - 3\text{i}$$

(*a*) Show that $z^2 = -5 - 12\text{i}$.

(2)

Find, showing your working,

(*b*) the value of $|z^2|$,

(2)

(*c*) the value of $\arg(z^2)$, giving your answer in radians to 2 decimal places.

(2)

(*d*) Show z and z^2 on a single Argand diagram.

(1)

June 2010

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3. (*a*) Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix},$$

find **AB**.

(2)

(*b*) Given that

$$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

and

$$\mathbf{E} = \mathbf{C} + \mathbf{D},$$

find the value of k for which **E** has no inverse.

(4)

June 2012

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4. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0,$$

(*a*) show that $2.2 < \alpha < 2.3$

(2)

(*b*) Taking 2.2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

(*c*) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

(3)

June 2009

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5. The parabola C has equation $y^2 = 20x$.

(*a*) Verify that the point $P(5t^2, 10t)$ is a general point on C.

(1)

The point A on C has parameter $t = 4$.
The line l passes through A and also passes through the focus of C.

(*b*) Find the gradient of l.

(4)

June 2010

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6. Given that $z = x + iy$, find the value of x and the value of y such that

$$z + 3iz^* = -1 + 13i$$

where z^* is the complex conjugate of z.

(7)

June 2011

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7. (*a*) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers n.

(6)

(*b*) Hence show that

$$\sum_{r=n+1}^{3n}(2r-1)^2 = \frac{2}{3}n(an^2+b)$$

where a and b are integers to be found.

(4)

June 2011

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8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point A on H has x-coordinate $3c$.

(*a*) Write down the y-coordinate of A.

(1)

(*b*) Show that an equation of the normal to H at A is

$$3y = 27x - 80c.$$

(5)

The normal to H at A meets H again at the point B.

(*c*) Find, in terms of c, the coordinates of B.

(5)

June 2010

9.

$$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}.$$

(*a*) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7,\ a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(*b*) Find the value of a.

(3)

The point R has coordinates (6, 0).

Given that O is the origin,

(*c*) find the area of triangle *ORS*.

(2)

Triangle *ORS* is mapped onto triangle *OR'S'* by the transformation represented by **M**.

(*d*) Find the area of triangle *OR'S'*.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(*e*) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

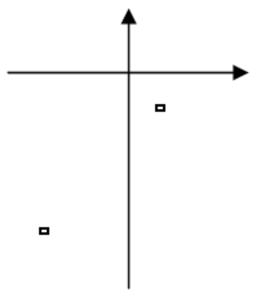
(*f*) Find **B**.

(4)

June 2012

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\mathrm{f}(x) = 3^x + 3x - 7$ $\mathrm{f}(1) = -1$ $\mathrm{f}(2) = 8$ **Sign change (positive, negative)** (and $\mathrm{f}(x)$ is continuous) therefore (a **root**) α is between $x = 1$ and $x = 2$.	M1 A1 (2)
(b)	$\mathrm{f}(1.5) = 2.696152423...$ $\{\Rightarrow 1 \leqslant \alpha \leqslant 1.5\}$ $\mathrm{f}(1.5) = \text{awrt } 2.7$ (or truncated to 2.6) $\mathrm{f}(1.25) = 0.698222038...$ $\Rightarrow 1 \leqslant \alpha \leqslant 1.25$	B1 M1 A1 (3) **[5]**
2. (a)	(2 – 3i)(2 – 3i) = ….. **Expand and use** $\mathrm{i}^2 = -1$, getting completely correct expansion of 3 or 4 terms Reaches –5 – 12i after completely correct work (must see 4 – 9) (*)	M1 A1cso (2)
(b)	$\lvert z^2 \rvert = \sqrt{(-5)^2 + (-12)^2} = 13$ or $\lvert z^2 \rvert = \sqrt{5^2 + 12^2} = 13$	M1 A1 (2)
(c)	$\tan \alpha = \frac{12}{5}$ (allow $-\frac{12}{5}$) or $\sin \alpha = \frac{12}{13}$ or $\cos \alpha = \frac{5}{13}$ $\arg(z^2) = -(\pi - 1.176\ldots) = -1.97$ (or 4.32) allow awrt	M1 A1 (2)
(d)	 Both in correct quadrants. Approximate relative scale No labels needed Allow two diagrams if some indication of scale Allow points or arrows	B1 (1) **[7]**

Question Number	Scheme	Marks
3. (a)	$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$	
	$= \begin{pmatrix} 3+1+0 & 3+2-3 \\ 4+5+0 & 4+10-5 \end{pmatrix}$	M1
	$= \begin{pmatrix} 4 & 2 \\ 9 & 9 \end{pmatrix}$	A1
		(2)
(b)	$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}$, where k is a constant,	
	$\mathbf{C} + \mathbf{D} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix} = \begin{pmatrix} 8 & 2k+2 \\ 12 & 6+k \end{pmatrix}$	M1
	E does not have an inverse $\Rightarrow$ $\det \mathbf{E} = 0$.	
	$8(6+k) - 12(2k+2)$	M1
	$8(6+k) - 12(2k+2) = 0$	M1
	$48 + 8k = 24k + 24$	
	$24 = 16k$	
	$k = \frac{3}{2}$	A1 oe
		(4) **[6]**
4. (a)	$\mathrm{f}(2.2) = 2.2^3 - 2.2^2 - 6 \quad (= -0.192)$	
	$\mathrm{f}(2.3) = 2.3^3 - 2.3^2 - 6 \quad (= 0.877)$	M1
	Change of sign $\Rightarrow$ Root need numerical values correct (to 1 s.f.).	A1
		(2)
(b)	$\mathrm{f}'(x) = 3x^2 - 2x$	B1
	$\mathrm{f}'(2.2) = 10.12$	B1
	$x_1 = x_0 - \frac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} = 2.2 - \frac{-0.192}{10.12}$	M1 A1ft
	$= 2.219$	A1cao
		(5)
(c)	$\frac{\alpha - 2.2}{\pm'0.192'} = \frac{2.3 - \alpha}{\pm'0.877'}$ (o.e. such as $\frac{k}{\pm'0.192'} = \frac{0.1-k}{\pm'0.877'}$.)	M1
	$\alpha(0.877 + 0.192) = 2.3 \times 0.192 + 2.2 \times 0.877$ or $k(0.877 + 0.192) = 0.1 \times 0.192$, where $\alpha = 2.2 + k$	A1
	so $\alpha \approx 2.218$ (2.21796…) (Allow awrt)	A1
		(3) **[10]**

Question Number	Scheme	Marks
5. (a)	$y^2 = (10t)^2 = 100t^2$ and $20x = 20\times5t^2 = 100t^2$	(1)
(b)	Point A is (80, 40) (stated or seen on diagram).	B1
	Focus is (5, 0) (stated or seen on diagram) or (a, 0) with a = 5.	B1
	Gradient: $\dfrac{40-0}{80-5} = \dfrac{40}{75}\left(=\dfrac{8}{15}\right)$	M1 A1
		(4) **[5]**
6. (a)	$z + 3\mathrm{i}z^* = -1 + 13\mathrm{i}$	
	$(x + \mathrm{i}y) + 3\mathrm{i}(x - \mathrm{i}y)$	B1 M1
	$x + \mathrm{i}y + 3\mathrm{i}x + 3y = -1 + 13\mathrm{i}$	A1
	$(x + 3y) + \mathrm{i}(y + 3x) = -1 + 13\mathrm{i}$	
	Re part: $x + 3y = -1$ Im part: $y + 3x = 13$	M1 A1
	$3x + 9y = -3$ $3x + y = 13$	
	$8y = -16 \Rightarrow y = -2$	M1
	$x + 3y = -1 \Rightarrow x - 6 = -1 \Rightarrow x = 5$	A1
	$\{z = 5 - 2\mathrm{i}\}$	
		[7]

Question Number	Scheme	Marks
7. (a)	$\{S_n =\} \sum_{r=1}^{n} (2r-1)^2$	
	$= \sum_{r=1}^{n} 4r^2 - 4r + 1$	M1 A1
	$= \underline{4.\frac{1}{6}n(n+1)(2n+1) - 4.\frac{1}{2}n(n+1) + n}$	B1
	$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$	
	$= \frac{1}{3}n\{2(n+1)(2n+1) - 6(n+1) + 3\}$	M1 A1
	$= \frac{1}{3}n\{2(2n^2+3n+1) - 6(n+1) + 3\}$	
	$= \frac{1}{3}n\{4n^2+6n+2 - 6n - 6 + 3\}$	
	$= \frac{1}{3}n(4n^2 - 1)$	
	$= \frac{1}{3}n(2n+1)(2n-1)$	A1 *
		(6)
(b)	$\sum_{r=n+1}^{3n} (2r-1)^2 = S_{3n} - S_n$	
	$= \frac{1}{3}.3n(6n+1)(6n-1) - \frac{1}{3}n(2n+1)(2n-1)$	M1 A1
	$= n(36n^2 - 1) - \frac{1}{3}n(4n^2 - 1)$	
	$= \frac{1}{3}n(108n^2 - 3 - 4n^2 + 1)$	dM1
	$= \frac{1}{3}n(104n^2 - 2)$	
	$= \frac{2}{3}n(52n^2 - 1)$	A1
	$\{a = 52,\ b = -1\}$	
		(4) **[10]**

Question Number	Scheme	Marks
8. (a)	$\dfrac{c}{3}$	B1
		(1)
(b)	$y=\dfrac{c^2}{x}\Rightarrow\dfrac{dy}{dx}=-c^2x^{-2}$,	B1
	or $y+x\dfrac{dy}{dx}=0\Rightarrow\dfrac{dy}{dx}=-\dfrac{y}{x}$ or $x=c, y=-\dfrac{c}{t^2}$ so $\dfrac{dy}{dx}=-\dfrac{1}{t^2}$	
	and at A $\dfrac{dy}{dx}=-\dfrac{c^2}{(3c)^2}=-\dfrac{1}{9}$ so gradient of normal is 9	M1 A1
	Either $y-\dfrac{c}{3}=9(x-3c)$ or $y=9x+k$ and use $x=3c,\ y=\dfrac{c}{3}$	M1
	$\Rightarrow$ $3y = 27x - 80c$ (*)	A1
		(5)
(c)	$\dfrac{c^2}{x}=\dfrac{27x-80c}{3}$ | $\dfrac{c^2}{y}=\dfrac{3y+80c}{27}$ | $3\dfrac{c}{t}=27ct-80c$	M1
	$3c^2 = 27x^2 - 80cx$ | $27c^2 = 3y^2 + 80cy$ | $3c = 27ct^2 - 80ct$	A1
	$(x - 3c)(27x + c) = 0$ | $(y + 27c)(3y - c) = 0$ | $(t - 3)(27t + 1) = 0$	M1
	$x=-\dfrac{c}{27}$, $y = -27c$ | $x=-\dfrac{c}{27}$, $y = -27c$ | ($t = -\dfrac{1}{27}$ and so) $x=-\dfrac{c}{27}$, $y = -27c$	A1, A1
		(5)
		[11]

Question Number	Scheme	Marks
9. (a)	$\det \mathbf{M} = 3(-5) - (4)(2) = -15 - 8 = \underline{-23}$	B1
		(1)
(b)	Therefore, $\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	M1
	Either, $3(2a-7) + 4(a-1) = 25$ or $2(2a-7) - 5(a-1) = -14$	
	or $\begin{pmatrix} 3(2a-7)+4(a-1) \\ 2(2a-7)-5(a-1) \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	A1
	giving $a = 5$	A1
		(3)
(c)	$\text{Area}(ORS) = \frac{1}{2}(6)(4);= \underline{12}$ (units)^2	M1A1
		(2)
(d)	$\text{Area}(OR'S') = \pm 23 \times (12)$	M1 A1 $\sqrt{\ }$
		(2)
(e)	Rotation; 90° anti-clockwise (or 270° clockwise) about $(0, 0)$.	B1;B1
		(2)
(f)	$\mathbf{M} = \mathbf{BA}$	M1
	$\mathbf{A}^{-1} = \frac{1}{(0)(0)-(1)(-1)}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A1
	$\mathbf{B} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	M1
	$\mathbf{B} = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$	A1
		(4) **[14]**

Examiner reports

Question 1

This question was well done by the vast majority of candidates. In part (a) virtually all successfully evaluated f(1) and f(2) and made an appropriate conclusion. There were a surprising number of cases where the conclusion was incomplete or omitted.

In part (b) again the work was often clear with many candidates using a table and making the correct conclusion. However candidates should be aware that values of the function are required e.g. in this case f(1.5) and f(1.25), to justify their conclusions. Again there were a surprising number of cases where the conclusion was omitted. Misinterpreting the requirement and applying Linear Interpolation was seen but was relatively rare.

Question 2

Most candidates found this question very accessible with many scoring 7 marks. In part (a) a minority of candidates failed to appreciate this was a "show that" question and omitted the key step of stating or using $i^2 = -1$. Other than this, numerical errors were very rare. Part (b) was very well done and the modulus was usually correctly given as 13. In part (c) most candidates appreciated that inverse tangent was needed, but many could not deal with the fact that the point was in the third quadrant. Wrong answers such as 1.97 and 1.18 were common. The need to identify the correct quadrant is essential for candidates hoping to continue to FP2 and FP3. In addition, a common error was to give the answer as -1.96, arising from rounding too early when finding $-\pi + 1.18$. In part (d) the Argand diagram was usually correct, though there were some errors. Candidates should be advised not to extend their working to the very bottom of the page, past the scanned area. Many plots of $-5 - 12i$ were beyond the scanned area.

Question 3

Part (a) was well answered with most candidates multiplying the matrices together correctly. Only a few candidates multiplied the matrices the wrong way round. In part (b) many could add the matrices correctly although a few candidates multiplied. The condition for there not being an inverse for **E** was well known and most attempted the determinant and set it to zero. The resulting equation in k was usually solved correctly although there were some basic algebraic errors.

Question 4

Most candidates were clear about the steps necessary to show that the root of the given equation was between the values 2.2 and 2.3 in part (a). Almost all substituted 2.2 and 2.3 into the left hand side of the equation and gave their numeric answers. A few did not complete the solution by stating that one answer was positive and one negative and that the sign change indicated the presence of a root between 2.2 and 2.3.

The Newton Raphson method in part (b) was well understood and most answered this part of the question correctly. Candidates are advised to show their expression for $f'(x)$ and for $f'(2.2)$. They are also advised to quote the formula and show their substitution. The final answer 2.219 was not acceptable with no working.

There were many good answers to part (c), with most solutions using similar triangles. Those who had learned and quoted a formula often made sign slips. Some used the equation of the line joining (2.2, -0.192) and (2.3, 0.877) and found where it crossed the *x* axis. This was an acceptable alternative method. A small number of candidates tried interval bisection however, which was not linear interpolation!

Question 5

In part (a) a variety of methods were used to verify that the given point was on the parabola, the most common being to substitute *x* or *y* into the equation. Many candidates instead found the value of '*a*' and substituted into the parametric equations of the parabola, unfortunately losing this mark if insufficient explanation was given e.g. the general parametric equations were not quoted. Part (b) was very well answered by most candidates. Most found the coordinates of the relevant points and used the gradient formula correctly. A surprising number quoted or used the gradient as $\frac{x_2 - x_1}{y_2 - y_1}$. The main error was to use differentiation to find the gradient of the line, an error which was penalised heavily as it was a complete misinterpretation of the question.

Question 6

Many candidates gained full marks in this question. It was pleasing to see that most knew what the complex conjugate was. Candidates who did struggle with this question were those who were unaware that following the substitution, there was a requirement to equate real and imaginary parts. There were some candidates who interpreted $z + 3\mathrm{i}z^*$ as $(z + 3\mathrm{i})z^*$.

Question 7

In part (a) virtually all candidates expanded correctly and substituted the standard formulae as well as identifying the "+*n*". Weaker candidates then often struggled with the resulting algebra. Candidates should be encouraged to use the printed result to identify potential factors if possible, rather than multiply out completely and then start working towards the result.

Part (b) was met with less success with a large number of candidates starting again from scratch by expanding the brackets rather than using the hence. Those who did use the result in part (a) often misinterpreted what was meant by S_{3n} and this sum often ended up as $3S_n$.

Question 8

Part (a) was extremely well done with very few errors. Several methods were used in (b) to find $\frac{dy}{dx}$, the most common being to differentiate c^2x^{-1}. Those who used implicit differentiation were also mostly successful as well as those who differentiated the parametric equations. Some simply quoted the derivative either in Cartesian or parametric form and these lost marks as the answer was given and this was a "show that" question. Most candidates however did show a clear, full solution to find the equation of the normal. There were some who failed to obtain a numerical value for the gradient so they ended up with a non-linear equation and this was penalised.

The algebra in part (c) was quite challenging for some but the standard of work was very pleasing. Most used simultaneous equations, one linear one the equation of the hyperbola. Usually candidates were able to eliminate one variable successfully to obtain a quadratic in one variable. They were then able to use a variety of methods to solve the quadratic. Many opted for the simple alternative of factorising, noticing that (x – 3c) was a factor. Those who used the quadratic formula sometimes confused their powers of c. The method of completing the square to solve this quadratic usually led to errors. A small number of candidates used an alternative approach to (c). They began with $\frac{\frac{c}{t}-\frac{c}{3}}{ct-3c}=9$ and simplified the fraction to give a linear equation leading to a value for t and then to x and y. On the whole, candidates showed a good understanding of the concepts required to tackle this question and were able to apply their skills successfully.

Question 9

Part (a) caused few problems and any errors were largely arithmetical.

Part (b) was met with a great deal of success and the vast majority could obtain the correct value for a. There were a few cases where candidates took the more laborious route of working out $\mathbf{M}^{-1}$ and worked 'backwards'.

In part (c) the area of *ORS* was often calculated correctly and in part (d) the determinant property for areas was well known and candidates could score a follow through mark for an incorrect determinant and/or *ORS* area.

Part (e) was disappointing in that although many candidates appreciated that the transformation was a rotation of 90° anticlockwise, they failed to give the centre. A few candidates thought the transformation was a reflection.

Part (f) was a good test of whether a candidate knew in which order to write the matrices, given the combination of two transformations. In fact there were possibly equal numbers of candidates who started with **M** = **BA** as those with **M** = **AB**. The subsequent use of $\mathbf{A}^{-1}$ was often correctly applied. Many candidates chose to represents **B** as a general matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and then calculate either **BA** or **AB** and compare the result with **M** to establish the values of a, b, c and d and hence the matrix **B**. This proved to be an efficient method given the nature of the matrix **A**. The incorrect matrix for **B** as $\begin{pmatrix} 2 & -5 \\ -3 & -4 \end{pmatrix}$ was common following an error with the order for matrix multiplication.

Statistics for FP1 Practice Paper Silver Level S1

Qu	Max Score	Modal score	Mean %	Mean score for students achieving grade: ALL	A*	A	B	C	D	E	U
1	5		89	4.46	4.93	4.80	4.49	4.32	4.05	3.84	3.07
2	7		89	6.20	6.72	6.52	6.25	6.04	5.88	5.61	4.71
3	6		86	5.14	5.94	5.72	5.35	4.99	4.52	4.11	2.98
4	10		89	8.90		9.67	9.03	8.61	8.09	6.99	5.71
5	5		88	4.39	4.96	4.80	4.54	4.27	3.95	3.46	2.39
6	7		80	5.61	6.90	6.62	5.82	5.06	4.23	3.76	2.36
7	10		77	7.74	9.89	9.25	7.90	6.67	5.37	4.63	2.71
8	11		77	8.45	10.81	10.21	8.89	7.98	6.37	5.04	3.27
9	14		70	9.81	13.24	12.22	10.16	8.43	6.43	4.99	3.01
	75		**81**	**60.70**		**69.81**	**62.43**	**56.37**	**48.89**	**42.43**	**30.21**