GCE Examinations

Decision Mathematics Module D2

Advanced Subsidiary / Advanced Level

Paper F

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 6 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Craig Hunter, Edited by Shaun Armstrong

1. A team of gardeners is called in to attend to the grounds of a stately home. The three gardeners will each be assigned to one of three areas, the lawns, the hedgerows and the flower beds. The table below shows the estimated time, in hours, it will take each gardener to do each job.

	Lawns	Hedgerows	Flower Beds
Alan	4	4.5	6
Beth	3	4	5
Colin	3.5	5	6

The team wishes to complete the tasks in the least total time.

Formulate this information as a linear programming problem.

(a) State your decision variables. **(2 marks)**

(b) Write down the objective function in terms of your decision variables. **(2 marks)**

(c) Write down the constraints and explain what each one represents. **(3 marks)**

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2. *This question should be answered on the sheet provided.*

A pool player is to play in four tournaments. Some of the tournaments take place simultaneously and the player has to choose one of each of the following:

1st tournament: *A*, *B* or *C*,
2nd tournament: *D*, *E* or *F*,
3rd tournament: *G*, *H* or *I*,
4th tournament: *J*, *K* or *L*.

Each tournament has six rounds and the player estimates how well he will do in each tournament based on which tournament he plays before it. The table below shows his expectations with each number indicating the round he expects to reach and a "7" indicating he expects to win the tournament.

		Expected performance in tournament											
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
Previous tournament	*None*	5	3	3									
	A				6	3	7						
	B				5	5	4						
	C				7	5	5						
	D							5	3	3			
	E							3	5	6			
	F							3	6	5			
	G										2	4	1
	H										3	2	2
	I										2	5	3

He wishes to choose the tournaments such that his worst performance is as good as possible. Use dynamic programming to find which tournaments he should play.

(10 marks)

Turn over

3. Four people are contributing to the entertainment section of an email magazine. For one issue reviews are required for a film, a musical, a ballet and a concert such that each person reviews one show. The people in charge of the magazine will pay each person's expenses and the cost, in pounds, for each reviewer to attend each show are given below.

	Film	Musical	Ballet	Concert
Andrew	5	20	12	18
Betty	6	18	15	16
Carlos	4	21	9	15
Davina	5	16	11	13

Use the Hungarian algorithm to find an optimal assignment which minimises the total cost. State the total cost of this allocation.

(10 marks)

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4. The payoff matrix for player A in a two-person zero-sum game is shown below.

		B	
		I	II
A	I	4	$^{-}8$
	II	2	$^{-}4$
	III	$^{-}8$	2

(a) Explain why the game does not have a saddle point. **(3 marks)**

(b) Using a graphical method, find the optimal strategy for player B. **(6 marks)**

(c) Find the optimal strategy for player A. **(4 marks)**

(d) Find the value of the game. **(2 marks)**

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5. A carpet manufacturer has two warehouses, W_1 and W_2, which supply carpets for three sales outlets, S_1, S_2 and S_3. At one point S_1 requires 40 rolls of carpet, S_2 requires 23 rolls of carpet and S_3 requires 37 rolls of carpet. At this point W_1 has 45 rolls in stock and W_2 has 40 rolls in stock. The following table shows the cost, in pounds, of transporting one roll from each warehouse to each sales outlet:

	S_1	S_2	S_3
W_1	8	7	11
W_2	9	10	11

The company's manager wishes to supply the 85 rolls that are in stock such that transportation costs are kept to a minimum.

(a) Use the north-west corner rule to obtain an initial solution to the problem. **(3 marks)**

(b) Calculate improvement indices for the unused routes. **(5 marks)**

(c) Use the stepping-stone method to obtain an optimal solution. **(8 marks)**

Turn over

6. *This question should be answered on the sheet provided.*

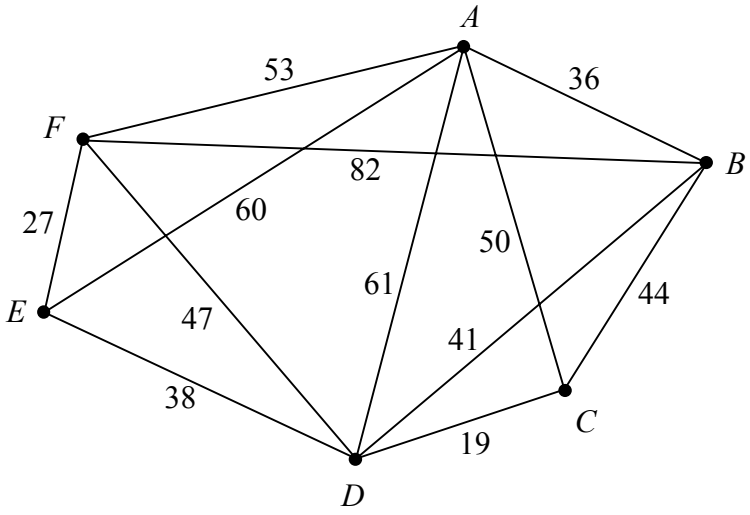

Fig. 1

A band is going on tour to play gigs in six towns, including their home town, *A*. The network in Figure 1 shows the distances, in miles, between the various towns. The band must begin and end their tour at *A* and visit each of the other towns once, and they wish to keep the total distance travelled as small as possible.

(a) By inspection, draw a complete network showing the shortest distances between the towns.

(3 marks)

(b) Use your complete network and the nearest neighbour algorithm, starting at *A*, to find an upper bound for the total distance travelled.

(3 marks)

(c) (i) Use your complete network to obtain and draw a minimum spanning tree and hence obtain another upper bound for the total distance travelled.

(ii) Improve this upper bound using two shortcuts to find an upper bound below 225 miles.

(7 marks)

(d) By deleting *A*, find a lower bound for the total distance travelled. **(3 marks)**

(e) State an interval of as small a width as possible within which *d*, the minimum distance travelled, in miles, must lie.

(1 mark)

END

Sheet for answering question 2 *NAME …….*

Please hand this sheet in for marking

Stage	Previous tournament	Current tournament	
1	*G*	*J* *K* *L*	
	H	*J* *K* *L*	
	I	*J* *K* *L*	
2	*D*	*G* *H* *I*	
	E	*G* *H* *I*	
	F	*G* *H* *I*	
3	*A*	*D* *E* *F*	
	B	*D* *E* *F*	
	C	*D* *E* *F*	
4	*None*	*A* *B* *C*	

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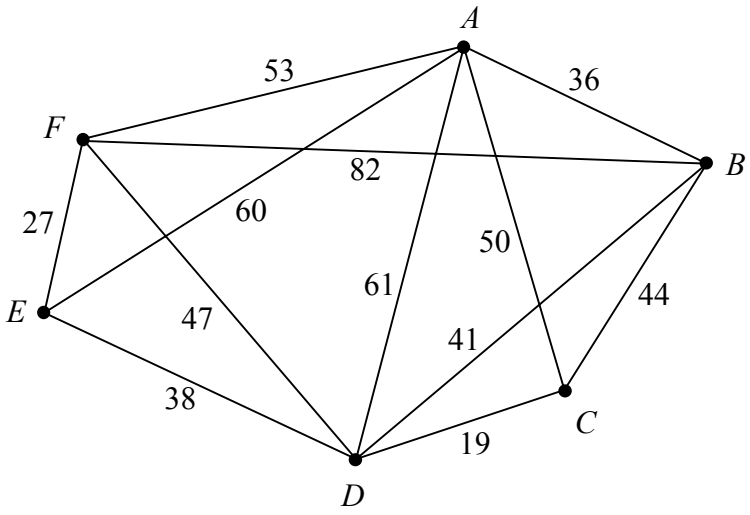
Sheet for answering question 6 *NAME* …….

Please hand this sheet in for marking

(a)



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(b)

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Turn over

Sheet for answering question 6 (cont.)

(c) (i)

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(ii) ……

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(d)

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(e) ……

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