

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper L

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



Written by Shaun Armstrong

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C4 Paper L – Marking Guide

1. (a) $\frac{dn}{dt} = 0 \Rightarrow e^{0.5t} = 5$ M1
 $t = 2 \ln 5 = 3.219 \text{ mins} = 3 \text{ mins } 13 \text{ secs}$ M1 A1
- (b) $\int dn = \int (e^{0.5t} - 5) dt$
 $n = 2e^{0.5t} - 5t + c$ M1 A1
 $t = 0, n = 20 \Rightarrow 20 = 2 + c, \quad c = 18$ M1
 $n = 2e^{0.5t} - 5t + 18$ A1
- (c) as t increases, n rapidly becomes very large \therefore not realistic B1 **(8)**
-
2. $6x + y + x \frac{dy}{dx} - 4y \frac{dy}{dx} = 0$ M1 A2
- $(1, 4) \Rightarrow 6 + 4 + \frac{dy}{dx} - 16 \frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{2}{3}$ M1 A1
- grad of normal = $-\frac{3}{2}$ M1
- $\therefore y - 4 = -\frac{3}{2}(x - 1)$ M1
- $2y - 8 = -3x + 3$
 $3x + 2y - 11 = 0$ A1 **(8)**
-
3. (a) $u = 2 - x^2 \Rightarrow \frac{du}{dx} = -2x$ M1
- $I = \int \frac{1}{u} \times (-\frac{1}{2}) du = -\frac{1}{2} \int \frac{1}{u} du$ A1
 $= -\frac{1}{2} \ln |u| + c = -\frac{1}{2} \ln |2 - x^2| + c$ M1 A1
- (b) $= \int_0^{\frac{\pi}{4}} (\frac{1}{2} \sin 4x + \frac{1}{2} \sin 2x) dx$ M1 A1
- $= [-\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x]_0^{\frac{\pi}{4}}$ M1 A1
- $= (\frac{1}{8} - 0) - (-\frac{1}{8} - \frac{1}{4}) = \frac{1}{2}$ M1 A1 **(10)**
-
4. (a) $\begin{matrix} x & 1 & 2 & 3 \\ y & 0 & 1.665 & 3.144 \end{matrix}$ B1
 $\text{area} \approx \frac{1}{2} \times 1 \times [0 + 3.144 + 2(1.665)] = 3.24 \text{ (3sf)}$ B1 M1 A1
- (b) $\text{volume} = \pi \int_1^3 x^2 \ln x \, dx$ M1
- $u = \ln x, \quad u' = \frac{1}{x}, \quad v' = x^2, \quad v = \frac{1}{3}x^3$
- $I = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 \, dx$ M1 A2
 $= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$ A1
- $\text{volume} = \pi [\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3]_1^3$
 $= \pi \{(9 \ln 3 - 3) - (0 - \frac{1}{9})\}$ M1
 $= \pi(9 \ln 3 - \frac{26}{9})$ A1 **(11)**
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5. (a) $\frac{5-8x}{(1+2x)(1-x)^2} \equiv \frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$
 $5-8x \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)(1-x)$ M1
 $x = -\frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$ A1
 $x = 1 \Rightarrow -3 = 3C \Rightarrow C = -1$ A1
 coeffs $x^2 \Rightarrow 0 = A - 2B \Rightarrow B = 2$ M1 A1
 $f(x) = \frac{4}{1+2x} + \frac{2}{1-x} - \frac{1}{(1-x)^2}$
- (b) $f(x) = 4(1+2x)^{-1} + 2(1-x)^{-1} - (1-x)^{-2}$
 $(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(2x)^3 + \dots$ M1
 $= 1 - 2x + 4x^2 - 8x^3 + \dots$ A1
 $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$ B1
 $(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{3 \times 2}(-x)^3 + \dots$
 $= 1 + 2x + 3x^2 + 4x^3 + \dots$ A1
 $f(x) = 4(1 - 2x + 4x^2 - 8x^3) + 2(1 + x + x^2 + x^3) - (1 + 2x + 3x^2 + 4x^3)$ M1
 $= 5 - 8x + 15x^2 - 34x^3 + \dots$ A1
- (c) $|x| < \frac{1}{2}$ A1 (12)

6. (a) $\frac{dx}{dt} = 1 + \cos t, \quad \frac{dy}{dt} = \cos t$ M1
 $\frac{dy}{dx} = \frac{\cos t}{1 + \cos t}$ M1 A1
- (b) $\frac{\cos t}{1 + \cos t} = 0, \quad \cos t = 0, \quad t = \frac{\pi}{2}$ M1 A1
 $\therefore (\frac{\pi}{2} + 1, 1)$ A1
- (c) $= \int_0^{\pi} \sin t \times (1 + \cos t) dt = \int_0^{\pi} (\sin t + \frac{1}{2} \sin 2t) dt$ M1 A1
 $= [-\cos t - \frac{1}{4} \cos 2t]_0^{\pi}$ M1 A1
 $= (1 - \frac{1}{4}) - (-1 - \frac{1}{4}) = 2$ M1 A1 (12)

7. (a) $\vec{AB} = (8\mathbf{j} - 6\mathbf{k}) - (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) = (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ M1
 $\therefore \mathbf{r} = (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) + \lambda(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ A1
- (b) $3 - 3\lambda = -2 + 7\mu \quad (1)$
 $6 + 2\lambda = 10 - 4\mu \quad (2)$
 $-8 + 2\lambda = 6 + 6\mu \quad (3)$ B1
 $(3) - (2): -14 = -4 + 10\mu, \mu = -1, \lambda = 4$ M1 A1
 check (1) $3 - 12 = -2 - 7, \text{ true } \therefore \text{intersect}$ B1
- (c) $\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) - (7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) \quad \therefore (-9, 14, 0)$ M1 A1
- (d) $\vec{OC} = [(-2 + 7\mu)\mathbf{i} + (10 - 4\mu)\mathbf{j} + (6 + 6\mu)\mathbf{k}]$
 $\vec{AC} = \vec{OC} - \vec{OA} = [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}]$ M1 A1
 $\therefore [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}] \cdot (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$ M1
 $15 - 21\mu + 8 - 8\mu + 28 + 12\mu = 0$ A1
 $\mu = 3 \quad \therefore \vec{OC} = (19\mathbf{i} - 2\mathbf{j} + 24\mathbf{k})$ M1 A1 (14)

Total (75)

