

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper K

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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C4 Paper K – Marking Guide

$$\begin{aligned}
 1. \quad &= \pi \int_1^3 \frac{(3x+1)^2}{x} \, dx && \text{M1} \\
 &= \pi \int_1^3 \frac{9x^2+6x+1}{x} \, dx = \int_1^3 (9x+6+\frac{1}{x}) \, dx && \text{A1} \\
 &= \pi \left[\frac{9}{2}x^2 + 6x + \ln|x| \right]_1^3 && \text{M1 A1} \\
 &= \pi \left\{ \left(\frac{81}{2} + 18 + \ln 3 \right) - \left(\frac{9}{2} + 6 + 0 \right) \right\} && \text{M1} \\
 &= \pi(48 + \ln 3) && \text{A1} \quad \mathbf{(6)}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (a) \quad &(1-3x)^{-2} = 1 + (-2)(-3x) + \frac{(-2)(-3)}{2}(-3x)^2 + \frac{(-2)(-3)(-4)}{3 \times 2}(-3x)^3 + \dots && \text{M1} \\
 &= 1 + 6x + 27x^2 + 108x^3 + \dots && \text{A3} \\
 (b) \quad &\left(\frac{2-x}{1-3x} \right)^2 = (2-x)^2(1-3x)^{-2} = (4-4x+x^2)(1+6x+27x^2+108x^3+\dots) && \text{M1} \\
 &= 4 + 24x + 108x^2 + 432x^3 - 4x - 24x^2 - 108x^3 + x^2 + 6x^3 + \dots && \text{A1} \\
 \therefore \text{ for small } x, &\left(\frac{2-x}{1-3x} \right)^2 = 4 + 20x + 85x^2 + 330x^3 && \text{A1} \quad \mathbf{(7)}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (a) \quad &\frac{7+3x+2x^2}{(1-2x)(1+x)^2} \equiv \frac{A}{1-2x} + \frac{B}{1+x} + \frac{C}{(1+x)^2} \\
 &7+3x+2x^2 \equiv A(1+x)^2 + B(1-2x)(1+x) + C(1-2x) \\
 x = \frac{1}{2} &\Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4 && \text{B1} \\
 x = -1 &\Rightarrow 6 = 3C \Rightarrow C = 2 && \text{B1} \\
 \text{coeffs } x^2 &\Rightarrow 2 = A - 2B \Rightarrow B = 1 && \text{M1} \\
 \therefore f(x) &= \frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2} && \text{A1} \\
 (b) \quad &= \int_1^2 \left(\frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2} \right) dx \\
 &= [-2 \ln|1-2x| + \ln|1+x| - 2(1+x)^{-1}]_1^2 && \text{M1 A3} \\
 &= (-2 \ln 3 + \ln 3 - \frac{2}{3}) - (0 + \ln 2 - 1) && \text{M1} \\
 &= -\ln 3 - \ln 2 + \frac{1}{3} = \frac{1}{3} - \ln 6 \quad [p = \frac{1}{3}, q = 6] && \text{M1 A1} \quad \mathbf{(11)}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad (a) \quad &4\lambda = 6 + 14\mu \quad (1) \\
 &-3 - 2\lambda = 3 + 2\mu \quad (2) && \text{B1} \\
 &(1) + 2 \times (2): -6 = 12 + 18\mu, \mu = -1, \lambda = -2 && \text{M1 A1} \\
 &\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} - 2 \begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \\ 1 \end{pmatrix} && \text{M1 A1} \\
 (b) \quad &a - (-5) = -3, \quad a = -8 && \text{M1 A1} \\
 (c) \quad &\cos \theta = \frac{|5 \times (-5) + 4 \times 14 + (-2) \times 2|}{\sqrt{25+16+4} \times \sqrt{25+196+4}} && \text{M1 A1} \\
 &= \frac{27}{\sqrt{45} \times 15} = \frac{9}{3\sqrt{5} \times 5} = \frac{3}{5\sqrt{5}} = \frac{3}{25}\sqrt{5} && \text{M1 A1} \quad \mathbf{(11)}
 \end{aligned}$$

5. (a) $2x - 4y - 4x \frac{dy}{dx} + 4y \frac{dy}{dx} = 0$ M1 A2
- $$\frac{dy}{dx} = \frac{2x-4y}{4x-4y} = \frac{x-2y}{2x-2y}$$
- M1 A1
- (b) grad = $\frac{3}{2}$ M1
- $$\therefore y - 2 = \frac{3}{2}(x - 1)$$
- M1
- $$2y - 4 = 3x - 3$$
- $$3x - 2y + 1 = 0$$
- A1
- (c) $\frac{x-2y}{2x-2y} = \frac{3}{2}$ M1
- $$2(x - 2y) = 3(2x - 2y), \quad y = 2x$$
- A1
- sub. $\Rightarrow x^2 - 8x^2 + 8x^2 = 1$ M1
- $$x^2 = 1, \quad x = 1 \text{ (at } P) \text{ or } -1$$
- A1
- $$\therefore Q(-1, -2)$$
- A1 (12)

6. (a) $\frac{dN}{dt} = kN$ B1
- (b) $\int \frac{1}{N} dN = \int k dt$ M1
- $$\ln |N| = kt + c$$
- M1 A1
- $$t = 0, N = N_0 \Rightarrow \ln |N_0| = c$$
- M1
- $$\ln |N| = kt + \ln |N_0|, \quad \ln \left| \frac{N}{N_0} \right| = kt$$
- M1
- $$\frac{N}{N_0} = e^{kt}, \quad N = N_0 e^{kt}$$
- A1
- (c) $2N_0 = N_0 e^{6k}$ M1
- $$k = \frac{1}{6} \ln 2 = 0.116 \text{ (3sf)}$$
- M1 A1
- (d) $10N_0 = N_0 e^{0.1155t}$ M1
- $$t = \frac{1}{0.1155} \ln 10 = 19.932 \text{ hours} = 19 \text{ hours } 56 \text{ mins}$$
- M1 A1 (13)

7. (a) $x + \frac{1}{x} = \sec \theta + \tan \theta + \frac{1}{\sec \theta + \tan \theta} = \frac{(\sec \theta + \tan \theta)^2 + 1}{\sec \theta + \tan \theta}$ M1
- $$= \frac{\sec^2 \theta + 2 \sec \theta \tan \theta + \tan^2 \theta + 1}{\sec \theta + \tan \theta} = \frac{2 \sec^2 \theta + 2 \sec \theta \tan \theta}{\sec \theta + \tan \theta}$$
- M1 A1
- $$= \frac{2 \sec \theta (\sec \theta + \tan \theta)}{\sec \theta + \tan \theta} = 2 \sec \theta$$
- M1 A1
- (b) $\frac{x^2+1}{x} = \frac{2}{\cos \theta} \Rightarrow \cos \theta = \frac{2x}{x^2+1}$ M1
- $$\frac{y^2+1}{y} = \frac{2}{\sin \theta} \Rightarrow \sin \theta = \frac{2y}{y^2+1} \quad \therefore \frac{4x^2}{(x^2+1)^2} + \frac{4y^2}{(y^2+1)^2} = 1$$
- M1 A1
- (c) $\frac{dx}{d\theta} = \sec \theta \tan \theta + \sec^2 \theta$ M1
- $$= \sec \theta (\tan \theta + \sec \theta) = \frac{x^2+1}{2x} \times x = \frac{1}{2}(x^2+1)$$
- M1 A1
- (d) $\frac{dy}{d\theta} = -\operatorname{cosec} \theta \cot \theta - \operatorname{cosec}^2 \theta$ M1
- $$= -\operatorname{cosec} \theta (\cot \theta + \operatorname{cosec} \theta) = -\frac{y^2+1}{2y} \times y = -\frac{1}{2}(y^2+1)$$
- A1
- $$\therefore \frac{dy}{dx} = -\frac{y^2+1}{x^2+1}$$
- M1 A1 (15)

Total (75)

