FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper J
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. The region bounded by the curve $y = x^2 - 2x$ and the x-axis is rotated through 2π radians about the x-axis.

Find the volume of the solid formed, giving your answer in terms of π. **(6)**

---

2. Use the substitution $u = 1 - x^{\frac{1}{2}}$ to find

$$\int \frac{1}{1-x^{\frac{1}{2}}}\ \mathrm{d}x.$$ **(6)**

---

3. A curve has the equation

$$2\sin 2x - \tan y = 0.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form in terms of x and y. **(5)**

(b) Show that the tangent to the curve at the point $(\frac{\pi}{6}, \frac{\pi}{3})$ has the equation

$$y = \tfrac{1}{2}x + \tfrac{\pi}{4}.$$ **(3)**

4.

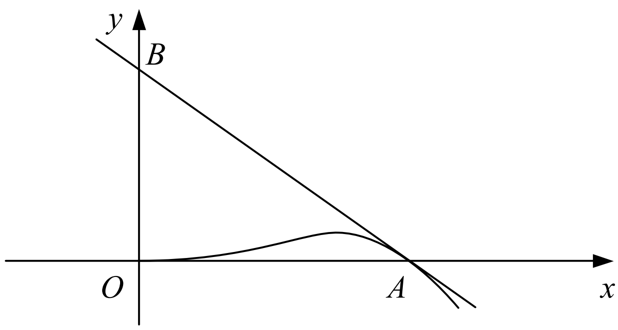

Figure 1

Figure 1 shows the curve with parametric equations

$$x = a\sqrt{t}, \quad y = at(1 - t), \quad t \geq 0,$$

where a is a positive constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. **(3)**

The curve meets the x-axis at the origin, O, and at the point A. The tangent to the curve at A meets the y-axis at the point B as shown.

(b) Show that the area of triangle OAB is a^2. **(6)**

---

5. The gradient at any point (x, y) on a curve is proportional to $\sqrt{y}$.

Given that the curve passes through the point with coordinates (0, 4),

(a) show that the equation of the curve can be written in the form

$$2\sqrt{y} = kx + 4,$$

where k is a positive constant. **(5)**

Given also that the curve passes through the point with coordinates (2, 9),

(b) find the equation of the curve in the form $y = \mathrm{f}(x)$. **(4)**

6.

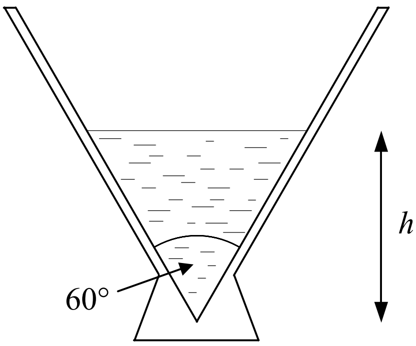

Figure 2

Figure 2 shows a vertical cross-section of a vase.

The inside of the vase is in the shape of a right-circular cone with the angle between the sides in the cross-section being 60°. When the depth of water in the vase is h cm, the volume of water in the vase is V cm^3.

(a) Show that $V = \frac{1}{9}\pi h^3$. **(3)**

The vase is initially empty and water is poured in at a constant rate of 120 cm^3 s^{-1}.

(b) Find, to 2 decimal places, the rate at which h is increasing

(i) when $h = 6$,

(ii) after water has been poured in for 8 seconds. **(7)**

Turn over

7. Relative to a fixed origin, the points A and B have position vectors $\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix}$ respectively.

(a) Find a vector equation for the line l_1 which passes through A and B. **(2)**

The line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ -7 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}.$$

(b) Show that lines l_1 and l_2 do not intersect. **(5)**

(c) Find the position vector of the point C on l_2 such that $\angle ABC = 90°$. **(6)**

8.

$$\text{f}(x) = \frac{x(3x-7)}{(1-x)(1-3x)}, \quad |x| < \tfrac{1}{3}.$$

(a) Find the values of the constants A, B and C such that

$$\text{f}(x) = A + \frac{B}{1-x} + \frac{C}{1-3x}.$$ **(4)**

(b) Evaluate

$$\int_0^{\frac{1}{4}} \text{f}(x) \ \text{d}x,$$

giving your answer in the form $p + \ln q$, where p and q are rational. **(5)**

(c) Find the series expansion of $\text{f}(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(5)**

END