FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper I

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. A curve has the equation

$$x^3 + 2xy - y^2 + 24 = 0.$$

Show that the normal to the curve at the point (2, –4) has the equation $y = 3x - 10$. **(8)**

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2. *(a)* Expand $(4 - x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2, simplifying each coefficient. **(4)**

(b) State the set of values of x for which your expansion is valid. **(1)**

(c) Use your expansion with $x = 0.01$ to find the value of $\sqrt{399}$, giving your answer to 9 significant figures. **(4)**

3.

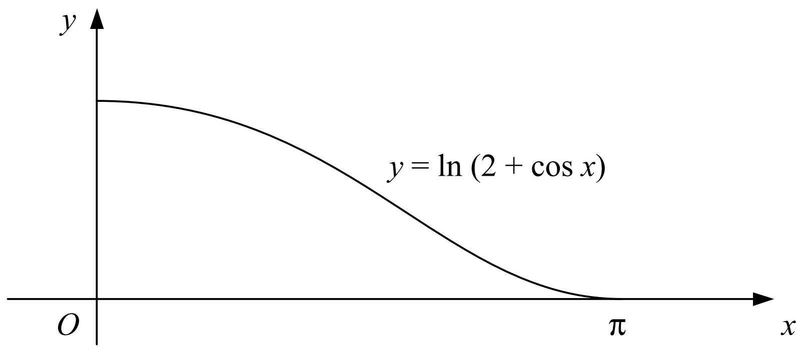

Figure 1

Figure 1 shows the curve with equation $y = \ln(2 + \cos x)$, $0 \le x \le \pi$.

(a) Copy and complete the table below for points on the curve, giving the y values to 4 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1.0986				0

(2)

(b) Giving your answers to 3 decimal places, find estimates for the area of the region bounded by the curve and the coordinate axes using the trapezium rule with

(i) 1 strip,

(ii) 2 strips,

(iii) 4 strips. . **(6)**

(c) Making your reasoning clear, suggest a value to 2 decimal places for the actual area of the region bounded by the curve and the coordinate axes. **(2)**

4.

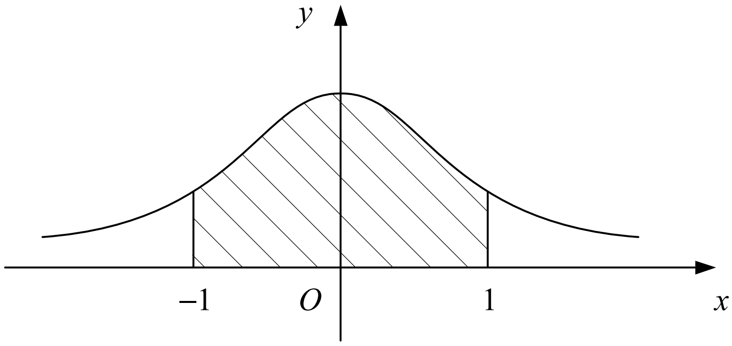

Figure 2

Figure 2 shows the curve with parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta, \quad -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}.$$

The shaded region bounded by the curve, the x-axis and the lines $x = -1$ and $x = 1$ is rotated through 2π radians about the x-axis.

(a) Show that the volume of the solid formed is $\frac{1}{4}\pi(\pi + 2)$. **(8)**

(b) Find a cartesian equation for the curve. **(3)**

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5. Relative to a fixed origin, the points A, B and C have position vectors $(2\mathbf{i} - \mathbf{j} + 6\mathbf{k})$, $(5\mathbf{i} - 4\mathbf{j})$ and $(7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k})$ respectively.

(a) Show that A, B and C all lie on a single straight line. **(3)**

(b) Write down the ratio $AB : BC$ **(1)**

The point D has position vector $(3\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

(c) Show that AD is perpendicular to BD. **(4)**

(d) Find the exact area of triangle ABD. **(3)**

Turn over

6. *(a)* Use the substitution $x = 2\sin u$ to evaluate

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\ \mathrm{d}x.$$ **(5)**

(b) Use integration by parts to evaluate

$$\int_0^{\frac{\pi}{2}} x\cos x\ \mathrm{d}x.$$ **(6)**

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7. When a plague of locusts attacks a wheat crop, the proportion of the crop destroyed after t hours is denoted by x. In a model, it is assumed that the rate at which the crop is destroyed is proportional to $x(1-x)$.

A plague of locusts is discovered in a wheat crop when one quarter of the crop has been destroyed.

Given that the rate of destruction at this instant is such that if it remained constant, the crop would be completely destroyed in a further six hours,

(a) show that $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2}{3}x(1-x)$, **(4)**

(b) find the percentage of the crop destroyed three hours after the plague of locusts is first discovered. **(11)**

END