GCE Examinations

Advanced Subsidiary

Core Mathematics C4

Paper G

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper G – Marking Guide

			Marks	Total
1.		$2x + 2y^2 + 2x \times 2y\dfrac{dy}{dx} + \dfrac{dy}{dx} = 0$	M2 A2	
		$\dfrac{dy}{dx} = -\dfrac{2x+2y^2}{4xy+1}$	M1 A1	**(6)**
2.		$u = x^2,\ u' = 2x,\ v' = e^{-x},\ v = -e^{-x}$	M1 A1	
		$I = -x^2 e^{-x} - \int -2x e^{-x}\ dx = -x^2 e^{-x} + \int 2x e^{-x}\ dx$	A2	
		$u = 2x,\ u' = 2,\ v' = e^{-x},\ v = -e^{-x}$	M1	
		$I = -x^2 e^{-x} - 2x e^{-x} - \int -2e^{-x}\ dx$	A1	
		$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c$	A1	**(7)**
3.	*(a)*	$(1 + ax)^n = 1 + nax + \frac{n(n-1)}{2}(ax)^2 + \ldots$	B1	
		$\therefore\ an = -4, \qquad \frac{a^2 n(n-1)}{2} = 24$	B1	
		$\Rightarrow a = \frac{-4}{n}$, sub. $\Rightarrow \frac{16}{n^2} \times \frac{n(n-1)}{2} = 24$	M1 A1	
		$8(n - 1) = 24n, \qquad n = -\frac{1}{2},\ a = 8$	M1 A1	
	(b)	$(1 + 8x)^{-\frac{1}{2}} = \ldots + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times2}(8x)^3 + \ldots$	M1	
		$\therefore\ k = -\frac{5}{16} \times 512 = -160$	A1	**(8)**
4.		x: 0, 0.75, 1.5, 2.25, 3 y: 2.7183, 2.0786, 1.0733, 0.5336, 0.3716	B2	
	(a)	$= \frac{1}{2} \times 1.5 \times [2.7183 + 0.3716 + 2(1.0733)] = 3.93$ (3sf)	B1 M1 A1	
	(b)	$= \frac{1}{2} \times 0.75 \times [2.7183 + 0.3716 + 2(2.0786 + 1.0733 + 0.5336)]$	M1	
		$= 3.92$ (3sf)	A1	
	(c)	curve must be above top of trapezia in some places and below in others hence position of ordinates determines whether estimate is high or low	B2	**(9)**
5.	*(a)*	$\overrightarrow{AB} = (4\mathbf{j} + \mathbf{k}) - (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) = (-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	M1	
		$\therefore\ \mathbf{r} = (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) + \lambda(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	A1	
		at C, $2 - \lambda = -1,\ \lambda = 3$	M1 A1	
		$\therefore\ \overrightarrow{OC} = (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) + 3(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k}) = (-18\mathbf{i} + 28\mathbf{j} - \mathbf{k})$	A1	
	(b)	$\overrightarrow{AC} = 3(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k}),\ AC = 3\sqrt{81+144+1} = 45.10$	M1 A1	
		$\therefore$ distance $= 200 \times 45.10 = 9020$ m $= 9.02$ km (3sf)	M1 A1	**(9)**

6.	*(a)*	$\int \frac{1}{P}\ \mathrm{d}P = \int 0.05\mathrm{e}^{-0.05t}\ \mathrm{d}t$	M1	
		$\ln\lvert P\rvert = -\mathrm{e}^{-0.05t} + c$	M1 A1	
		$t = 0,\ P = 9000 \Rightarrow \ln 9000 = -1 + c, \qquad c = 1 + \ln 9000$	M1	
		$\ln\lvert P\rvert = 1 + \ln 9000 - \mathrm{e}^{-0.05t}$	A1	
		$t = 10 \Rightarrow \ln\lvert P\rvert = 1 + \ln 9000 - \mathrm{e}^{-0.5} = 9.498$	M1	
		$P = \mathrm{e}^{9.498} = 13339 = 13300$ (3sf)	A1	
	(b)	$t \to \infty,\ \ln\lvert P\rvert \to 1 + \ln 9000$	M1	
		$\therefore\ P \to \mathrm{e}^{1 + \ln 9000} = 9000\mathrm{e} = 24465 = 24500$ (3sf)	M1 A1	**(10)**
7.	*(a)*	$x = 2 \Rightarrow t = 1, \quad x = 9 \Rightarrow t = 2$	B1	
		$\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2$	M1	
		$\therefore$ area $= \int_1^2 \frac{2}{t} \times 3t^2\ \mathrm{d}t = \int_1^2 6t\ \mathrm{d}t$	A1	
		$= [3t^2]_1^2 = 3(4 - 1) = 9$	M1 A1	
	(b)	$= \pi\int_1^2 (\frac{2}{t})^2 \times 3t^2\ \mathrm{d}t = \pi\int_1^2 12\ \mathrm{d}t$	M1	
		$= \pi[12t]_1^2 = 12\pi(2 - 1) = 12\pi$	M1 A1	
	(c)	$t = \frac{2}{y} \quad \therefore x = (\frac{2}{y})^3 + 1 = \frac{8}{y^3} + 1$	M1	
		$\therefore\ y^3 = \frac{8}{x-1}, \qquad y = \frac{2}{\sqrt[3]{x-1}}$	M1 A1	**(11)**
8.	*(a)*	$u = \sin x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \cos x$	B1	
		$I = \int \frac{6\cos x}{\cos^2 x(2-\sin x)}\ \mathrm{d}x = \int \frac{6\cos x}{(1-\sin^2 x)(2-\sin x)}\ \mathrm{d}x$	M1	
		$= \int \frac{6}{(1-u^2)(2-u)}\ \mathrm{d}u$	M1 A1	
	(b)	$\frac{6}{(1+u)(1-u)(2-u)} \equiv \frac{A}{1+u} + \frac{B}{1-u} + \frac{C}{2-u}$		
		$6 \equiv A(1 - u)(2 - u) + B(1 + u)(2 - u) + C(1 + u)(1 - u)$	M1	
		$u = -1 \Rightarrow 6 = 6A \Rightarrow A = 1$	A1	
		$u = 1 \Rightarrow 6 = 2B \Rightarrow B = 3$	A1	
		$u = 2 \Rightarrow 6 = -3C \Rightarrow C = -2$	A1	
		$\therefore\ \frac{6}{(1-u^2)(2-u)} \equiv \frac{1}{1+u} + \frac{3}{1-u} - \frac{2}{2-u}$		
	(c)	$x = 0 \Rightarrow u = 0,\ x = \frac{\pi}{6} \Rightarrow u = \frac{1}{2}$	M1	
		$I = \int_0^{\frac{1}{2}} (\frac{1}{1+u} + \frac{3}{1-u} - \frac{2}{2-u})\ \mathrm{d}u$		
		$= [\ln\lvert 1 + u\rvert - 3\ln\lvert 1 - u\rvert + 2\ln\lvert 2 - u\rvert]_0^{\frac{1}{2}}$	M1 A2	
		$= (\ln\frac{3}{2} - 3\ln\frac{1}{2} + 2\ln\frac{3}{2}) - (0 + 0 + 2\ln 2)$	M1	
		$= 3\ln\frac{3}{2} + 3\ln 2 - 2\ln 2$		
		$= 3\ln 3 - 3\ln 2 + \ln 2 = 3\ln 3 - 2\ln 2$	M1 A1	**(15)**
			Total	**(75)**

Performance Record – C4 Paper G

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	differentiation	integration	binomial series	trapezium rule	vectors	differential equation	parametric equations	partial fractions, integration	
Marks	6	7	8	9	9	10	11	15	75
Student									