FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper F

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Solomon Press

Written by Shaun Armstrong

1. A curve has the equation

$$2x^2 + xy - y^2 + 18 = 0.$$

Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. **(8)**

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2. Use the substitution $x = 2\tan u$ to show that

$$\int_0^2 \frac{x^2}{x^2+4}\, dx = \tfrac{1}{2}(4 - \pi).$$ **(8)**

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3. *(a)* Show that $(1\tfrac{1}{24})^{-\frac{1}{2}} = k\sqrt{6}$, where k is rational. **(2)**

(b) Expand $(1 + \tfrac{1}{2}x)^{-\frac{1}{2}}$, $|x| < 2$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4)**

(c) Use your answer to part *(b)* with $x = \tfrac{1}{12}$ to find an approximate value for $\sqrt{6}$, giving your answer to 5 decimal places. **(3)**

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4. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = (7\mathbf{j} - 4\mathbf{k}) + s(4\mathbf{i} - 3\mathbf{j} + \mathbf{k}),$$

and $$\mathbf{r} = (-7\mathbf{i} + \mathbf{j} + 8\mathbf{k}) + t(-3\mathbf{i} + 2\mathbf{k}),$$

where s and t are scalar parameters.

(a) Show that the two lines intersect and find the position vector of the point where they meet. **(5)**

(b) Find, in degrees to 1 decimal place, the acute angle between the lines. **(4)**

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5. A curve has parametric equations

$$x = \frac{t}{2-t}, \quad y = \frac{1}{1+t}, \quad -1 < t < 2.$$

(a) Show that $\frac{dy}{dx} = -\frac{1}{2}\left(\frac{2-t}{1+t}\right)^2$. **(4)**

(b) Find an equation for the normal to the curve at the point where $t = 1$. **(3)**

(c) Show that the cartesian equation of the curve can be written in the form

$$y = \frac{1+x}{1+3x}.$$ **(4)**

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6. *(a)* Find $\int \tan^2 x \; dx$. **(3)**

(b) Show that

$$\int \tan x \; dx = \ln |\sec x| + c,$$

where c is an arbitrary constant. **(4)**

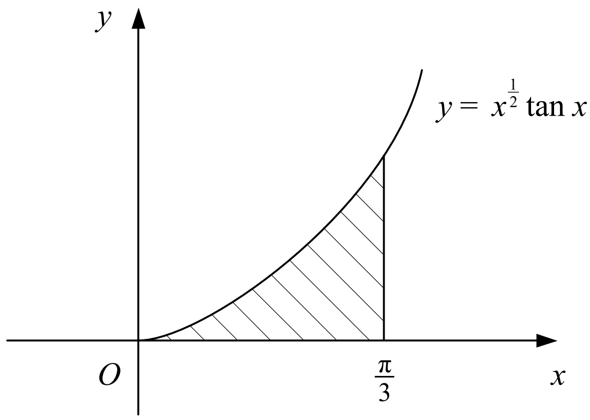

Figure 1

Figure 1 shows part of the curve with equation $y = x^{\frac{1}{2}} \tan x$.

The shaded region bounded by the curve, the x-axis and the line $x = \frac{\pi}{3}$ is rotated through 2π radians about the x-axis.

(c) Show that the volume of the solid formed is $\frac{1}{18}\pi^2(6\sqrt{3} - \pi) - \pi \ln 2$. **(6)**

Turn over

7.

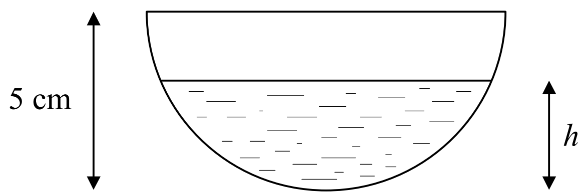

Figure 2

Figure 2 shows a hemispherical bowl of radius 5 cm.

The bowl is filled with water but the water leaks from a hole at the base of the bowl. At time t minutes, the depth of water is h cm and the volume of water in the bowl is V cm^3, where

$$V = \tfrac{1}{3}\pi h^2(15 - h).$$

In a model it is assumed that the rate at which the volume of water in the bowl decreases is proportional to V.

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{kh(15-h)}{3(10-h)},$$

where k is a positive constant. **(5)**

(b) Express $\dfrac{3(10-h)}{h(15-h)}$ in partial fractions. **(3)**

Given that when $t = 0$, $h = 5$,

(c) show that

$$h^2(15 - h) = 250\,\mathrm{e}^{-kt}.$$ **(6)**

Given also that when $t = 2$, $h = 4$,

(d) find the value of k to 3 significant figures. **(3)**

END