FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C4

Paper C

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. Use integration by parts to show that

$$\int_1^2 x \ln x \, dx = 2 \ln 2 - \tfrac{3}{4}. \quad \textbf{(6)}$$

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2. *(a)* Use the trapezium rule with two intervals of equal width to find an approximate value for the integral

$$\int_0^2 \arctan x \, dx. \quad \textbf{(5)}$$

(b) Use the trapezium rule with four intervals of equal width to find an improved approximation for the value of the integral. **(2)**

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3. A curve has the equation

$$3x^2 - 2x + xy + y^2 - 11 = 0.$$

The point P on the curve has coordinates $(-1, 3)$.

(a) Show that the normal to the curve at P has the equation $y = 2 - x$. **(7)**

(b) Find the coordinates of the point where the normal to the curve at P meets the curve again. **(4)**

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4. The points A and B have coordinates $(3, 9, -7)$ and $(13, -6, -2)$ respectively.

(a) Find, in vector form, an equation for the line l which passes through A and B. **(2)**

(b) Show that the point C with coordinates $(9, 0, -4)$ lies on l. **(2)**

The point D is the point on l closest to the origin, O.

(c) Find the coordinates of D. **(4)**

(d) Find the area of triangle OAB to 3 significant figures. **(3)**

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5. A bath is filled with hot water which is allowed to cool. The temperature of the water is θ°C after cooling for t minutes and the temperature of the room is assumed to remain constant at 20°C.

Given that the rate at which the temperature of the water decreases is proportional to the difference in temperature between the water and the room,

(a) write down a differential equation connecting θ and t. **(2)**

Given also that the temperature of the water is initially 37°C and that it is 36°C after cooling for four minutes,

(b) find, to 3 significant figures, the temperature of the water after ten minutes. **(8)**

Advice suggests that the temperature of the water should be allowed to cool to 33°C before a child gets in.

(c) Find, to the nearest second, how long a child should wait before getting into the bath. **(3)**

Turn over

6.

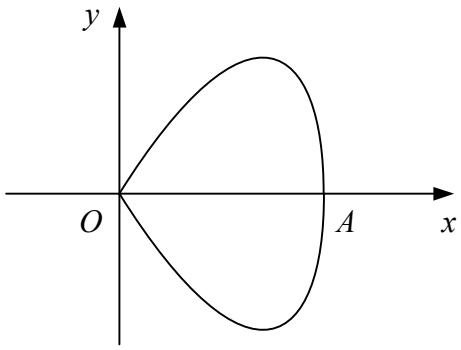

Figure 1

Figure 1 shows the curve with parametric equations

$$x = 3\sin t, \quad y = 2\sin 2t, \quad 0 \leq t < \pi.$$

The curve meets the x-axis at the origin, O, and at the point A.

(a) Find the value of t at O and the value of t at A. **(2)**

The region enclosed by the curve is rotated through π radians about the x-axis.

(b) Show that the volume of the solid formed is given by

$$\int_0^{\frac{\pi}{2}} 12\pi \sin^2 2t \cos t \, \mathrm{d}t. \quad \textbf{(3)}$$

(c) Using the substitution $u = \sin t$, or otherwise, evaluate this integral, giving your answer as an exact multiple of π. **(8)**

7.

$$\mathrm{f}(x) = \frac{8-x}{(1+x)(2-x)}, \quad |x| < 1.$$

(a) Express f(x) in partial fractions. **(3)**

(b) Show that

$$\int_0^{\frac{1}{2}} \mathrm{f}(x) \, \mathrm{d}x = \ln k,$$

where k is an integer to be found. **(5)**

(c) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(6)**

END