Worked Solutions

Edexcel C4 Paper A

1. (*a*) $\dfrac{3x}{(x-1)(x+2)} = \dfrac{1}{x-1} + \dfrac{2}{x+2}$ (using 'cover up' rule) **(3)**

(*b*) $\displaystyle\int_2^3 \left(\frac{1}{x-1} + \frac{2}{x+2}\right) dx = \Big[\ln(x-1) + 2\ln(x+2)\Big]_2^3$

$= \ln 2 + 2\ln 5 - (\ln 1 + 2\ln 4)$

$= \ln 2 + 2\ln \dfrac{5}{4}$

$= \ln 2 + \ln \dfrac{25}{16}$

$= \ln \dfrac{25}{8}$ **(4)**

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2. (*a*) $x = 1 - t^3$ and $x = 2$

$\therefore \quad 1 - t^3 = 2$

$t^3 = -1$

$t = -1$ **(1)**

(*b*) $\dfrac{dy}{dt} = 2t, \quad \dfrac{dx}{dt} = -3t^2$

$\dfrac{dy}{dx} = \dfrac{2t}{-3t^2} = \dfrac{-2}{3t}$

when $t = -1$, gradient of tangent $= \dfrac{2}{3}$.

equation of tangent is $y - 2 = \dfrac{2}{3}(x-2)$

$3y = 2x + 2$ **(4)**

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3. (*a*) $\dfrac{dy}{dx} = e^x - 3$

at M $\quad e^x = 3$

$x = \ln 3$ **(2)**

(*b*) area $= \displaystyle\int_0^{\ln 3} (e^x - 3x)dx = \left[e^x - \frac{3}{2}x^2\right]_0^{\ln 3}$

$= e^{\ln 3} - \dfrac{3}{2}(\ln 3)^2 - (1 - 0)$

$= 3 - \dfrac{3}{2}(\ln 3)^2 - 1$

$= 2 - \dfrac{3}{2}(\ln 3)^2$ **(5)**

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4. (*a*) $8x + 6y\dfrac{dy}{dx} - \left(2x\dfrac{dy}{dx} + y.2\right) = 0$

$\dfrac{dy}{dx}(6y - 2x) = 2y - 8x$

$\dfrac{dy}{dx} = \dfrac{2y - 8x}{6y - 2x} = \dfrac{y - 4x}{3y - x}$ **(6)**

(*b*) at $(2, 4)$, $\dfrac{dy}{dx} = \dfrac{4-8}{12-2} = -\dfrac{2}{5}$

equation of tangent at $(2, 4)$ is

$y - 4 = -\dfrac{2}{5}(x - 2)$

$5y - 20 = -2x + 4$

$5y + 2x = 24$ **(3)**

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5. (*a*) $(1+kx)^n = 1 + nkx + \frac{n(n-1)}{2}\cdot k^2x^2 + \frac{n(n-1)(n-2)}{3.2}\cdot k^3x^3 + \cdots$

$nk = -6$ ...[A]

$\frac{n(n-1)}{2}k^2 = 27$ ...[B]

from [A] $k = \frac{-6}{n}$.

substitute in [B] $\frac{n(n-1)}{2}\left(\frac{-6}{n}\right)^2 = 27$

hence $n = -2$ and $k = 3$ **(4)**

(*b*) coef. of $x^3 = \frac{-2.-3.-4}{3.2}.27 = -108$ **(3)**

(*c*) valid for $-1 < 3x < 1$

i.e. $-\frac{1}{3} < x < \frac{1}{3}$ **(1)**

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6. (*a*) Separating the variables, $y^{-2}\mathrm{d}y = \frac{4x^5-1}{x^2}\mathrm{d}x$

$\int y^{-2}\ \mathrm{d}y = \int \left(4x^3 - x^{-2}\right)\mathrm{d}x$

$-\frac{1}{y} = x^4 + \frac{1}{x} + c$

$y = \frac{1}{2}, x = 1 \Rightarrow -2 = 1 + 1 + c$ $\quad c = -4$

$\therefore\quad -\frac{1}{y} = x^4 + \frac{1}{x} - 4$ **(7)**

or $\quad -\frac{1}{y} = \frac{x^5+1-4x}{x}$

$y = -\left(\frac{x}{x^5+1-4x}\right)$

(*b*) Let $I = \int_0^2 \frac{x^3}{(1+x^2)^{\frac{1}{2}}}\mathrm{d}x$ $\qquad$ Let $t = 1 + x^2$, $\frac{\mathrm{d}t}{\mathrm{d}x} = 2x$

$\frac{1}{2}\mathrm{d}t = x\mathrm{d}x$

$\therefore\quad I = \frac{1}{2}\int_1^5 \frac{(t-1)\mathrm{d}t}{t^{\frac{1}{2}}}$

when $x = 0,\quad t = 1$
$x = 2,\quad t = 5$

$= \frac{1}{2}\int_1^5 \left(t^{\frac{1}{2}} - t^{-\frac{1}{2}}\right)\mathrm{d}t$

$= \frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right]_1^5 = \frac{1}{2}\left[\frac{2}{3}.5\sqrt{5} - 2\sqrt{5} - \left(\frac{2}{3} - 2\right)\right]$

$= \frac{1}{2}\left[\frac{10\sqrt{5} - 6\sqrt{5} - 2 + 6}{3}\right]$

$= \frac{1}{6}\left[4\sqrt{5}+4\right] = \frac{2}{3}\left(1+\sqrt{5}\right)$ **(7)**

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7. (*a*) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2 - \left(x.\frac{1}{x} + \ln x\right) = 1 - \ln x$

at Q $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \quad \therefore \quad \ln x = 1 \quad x = \mathrm{e}$

at $x = \mathrm{e}$, $\quad y = 2\mathrm{e} - \mathrm{e}\ln \mathrm{e} = \mathrm{e}$

Q is at (e, e)

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{1}{x}$, $\quad$ so $\frac{\mathrm{d}^2y}{\mathrm{d}x} < 0$ $\quad$ at $\quad x = \mathrm{e}$ **(4)**

(*b*) at P $\quad 2x - x\ln x = 0$

$x(2 - \ln x) = 0$

$\ln x = 2$

$x = \mathrm{e}^2$

coordinates of P are $(\mathrm{e}^2, 0)$ **(2)**

(c) (i) $\int_1^{e^2} x \ln x \ dx = \int_1^{e^2} \ln x \frac{d}{dx}\left(\frac{1}{2}x^2\right) dx$ [By parts]

$$= \left[\frac{1}{2}x^2 \ln x\right]_1^{e^2} - \int_1^{e^2} \frac{1}{2}x^2 . \frac{1}{x} \ dx$$

$$= \left[\frac{1}{2}x^2 \ln x\right]_1^{e^2} - \left[\frac{1}{4}x^2\right]_1^{e^2}$$

$$= \frac{1}{2}e^4 \ln e^2 - \frac{1}{2}1^2 . \ln 1 - \left[\frac{1}{4}e^4 - \frac{1}{4}\right]$$

$$= \frac{1}{\cancel{2}}e^4 \ \cancel{2} \ln e - 0 - \frac{1}{4}e^4 + \frac{1}{4}$$

$$= \frac{3e^4 + 1}{4} \qquad (\ln e = 1)$$ **(6)**

(ii) shaded area $= \int_1^{e^2} (2x - x \ln x) dx = \int_1^{e^2} 2x \ dx - \int_1^{e^2} x \ln x \ dx$

$$= \left[x^2\right]_1^{e^2} - \left(\frac{3e^4 + 1}{4}\right)$$

$$= e^4 - 1 - \left(\frac{3e^4 + 1}{4}\right)$$

$$= \frac{4e^4 - 4 - 3e^4 - 1}{4} = \frac{e^4 - 5}{4}$$ **(3)**

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8. (a) $\overrightarrow{BC} = \begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix}$

l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix}$ **(2)**

(b) $\overrightarrow{AD} = \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix}$, l_2 is $\mathbf{r} = \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix}$ **(2)**

(c) At point of intersection
$2 + 7\lambda = 6 - 2\mu$ ...[A]
$4 - \lambda = 2 + 6\mu$ ...[B]
$1 - \lambda = 0 + 2\mu$ ...[C]

from [A] and [B] $\lambda = \frac{1}{2}$ and $\mu = \frac{1}{4}$

check in [C] $1 - \frac{1}{2} = 2.\frac{1}{4}$

l_1 and l_2 intersect at $\left(5\frac{1}{2}, 3\frac{1}{2}, \frac{1}{2}\right)$. **(4)**

(d) we require the angle between $\begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix}$

let angle between lines be θ

$$\begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix} = \sqrt{7^2 + 1^2 + 1^2} \times \sqrt{2^2 + 6^2 + 2^2} \quad \cos\theta$$

$$-14 - 6 - 2 = \sqrt{51}\sqrt{44} \quad \cos\theta$$

$$\cos\theta = \frac{-22}{\sqrt{51}\sqrt{44}} \qquad \theta = 117.7°$$

acute angle between lines $= 62.3°$ (1 d.p.) **(3)**
