

Worked Solutions

Edexcel C4 Paper K

$$1. (a) V = \frac{4}{3}\pi r^3, \frac{dV}{dr} = 4\pi r^2 \quad (1)$$

$$(b) \frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$\begin{aligned} \text{given } r = 10 \text{ and } \frac{dr}{dt} = 0.1, \frac{dV}{dt} &= 4\pi \times 10^2 \times 0.1 \\ &= 40\pi \text{ cm}^3 \text{ s}^{-1} \end{aligned} \quad (4)$$

$$2. (a) 2x + x \cdot \frac{1}{y} \frac{dy}{dx} + \ln y + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \left(\frac{x}{y} + 1 \right) = -(2x + \ln y)$$

$$\frac{dy}{dx} = - \frac{(2x + \ln y)}{\left(\frac{x}{y} + 1 \right)}$$

$$\text{at } (3, 1) \text{ gradient} = - \left(\frac{6 + 0}{3 + 1} \right) = -\frac{3}{2} \quad (3)$$

$$(b) 6x + 2x \frac{dy}{dx} + y \cdot 2 - 10y \frac{dy}{dx} + 16 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (2x - 10y + 16) = -(6x + 2y)$$

$$\frac{dy}{dx} = 0 \text{ when } 6x + 2y = 0$$

$$\text{or } y = -3x$$

substitute $y = -3x$ into equation of curve,

$$3x^2 + 2x(-3x) - 5(9x^2) + 16(-3x) = 0$$

$$-48x^2 - 48x = 0$$

$$-48x(x + 1) = 0$$

$$x = 0 \text{ or } -1 \quad (5)$$

$$3. (a) \text{ at } P \ y = 0.$$

$$\cos t = 0$$

$$t = \frac{\pi}{2}$$

$$\text{at } t = \frac{\pi}{2}, x = \frac{\pi^2}{4}$$

$$\text{coordinates of } P \text{ are } \left(\frac{\pi^2}{4}, 0 \right) \quad (2)$$

$$(b) (i) \text{ area} = \int_y \frac{dx}{dt} dt = \int_0^{\frac{\pi}{2}} \cos t \cdot 2t dt \quad (2)$$

$$(ii) A = \int_0^{\frac{\pi}{2}} 2t \frac{d}{dt}(\sin t) dt \quad (\text{By parts})$$

$$= \left[2t \sin t \right]_0^{\frac{\pi}{2}} - 2 \int_0^{\frac{\pi}{2}} \sin t dt$$

$$= \left[2t \sin t + 2 \cos t \right]_0^{\frac{\pi}{2}}$$

$$= \pi + 0 - (0 + 2) = \pi - 2 \quad (5)$$

4. (a) $f(x) = (1 - 9x^2)^{-\frac{1}{2}}$

$$= 1 + \left(-\frac{1}{2}\right)(-9x^2) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-9x^2)^2$$

$$= 1 + \frac{9}{2}x^2 + \frac{243}{8}x^4$$

(b) valid for $|9x^2| < 1$

$$|x^2| < \frac{1}{9}, |x| < \frac{1}{3}$$

(c) (i) $\frac{(1+3x)^{\frac{1}{2}}}{(1-3x)^{\frac{1}{2}}} \times \frac{(1+3x)^{\frac{1}{2}}}{(1+3x)^{\frac{1}{2}}} = \frac{1+3x}{\sqrt{(1-9x^2)}}$

(ii) $(1+3x)\left(1 + \frac{9}{2}x^2 + \frac{243}{8}x^4\right)$

$$= 1 + \frac{9}{2}x^2 + \frac{243}{8}x^4 + 3x + \frac{27}{2}x^3 + \frac{729}{8}x^5$$

$$= 1 + 3x + \frac{9}{2}x^2 + \frac{27}{2}x^3 + \frac{243}{8}x^4 + \frac{729}{8}x^5$$

5. (a) $A = 160, B = 50$

[N doubles as t increases by 10]

(b) (i) $t = 10, m = 500e^{-1} = 184$

(ii) $300 = 500e^{-0.1t}$

$$\ln \frac{3}{5} = -0.1t$$

$$t = 5.1$$

(iii) $\frac{dm}{dt} = 500(-0.1)e^{-0.1t}$

when $t = 20, \frac{dm}{dt} = -6.77$ gram/year

6. (a) (i) $\int x \ln x \, dx = \int \ln x \frac{d}{dx} \left(\frac{x^2}{2}\right) dx$ (By parts)

$$= \frac{x^2}{2} \ln x - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx$$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$$

(ii) $\int \ln x \, dx = \int \ln x \frac{d}{dx}(x) \, dx$

$$= x \ln x - \int \frac{1}{x} \cdot x \, dx$$

$$= x \ln x - x + c$$

(b) let $I = \int_1^{-2} x\sqrt{x+3} \, dx$

let $u = x + 3$
 $\frac{du}{dx} = 1$

$$\therefore I = \int_4^1 (u-3)u^{\frac{1}{2}} \, du$$

when $x = -2, u = 1$
 $x = 1, u = 4$

$$= \int_4^1 \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right) \, du$$

$$= \left[\frac{2}{5}u^{\frac{5}{2}} - 2u^{\frac{3}{2}}\right]_4^1 = \frac{8}{5}$$

$$7. (a) \frac{dV}{dt} = 10 - \frac{V}{4}$$

$$4\frac{dV}{dt} = 40 - V, -4\frac{dV}{dt} = V - 40 \quad (3)$$

$$(b) \int \frac{1}{V-40} dV = -\frac{1}{4} \int dt$$

$$\ln(V-40) = -\frac{1}{4}t + c$$

$$V = 100, t = 0: \ln 60 = 0 + c$$

$$\therefore \ln(V-40) = -\frac{1}{4}t + \ln 60$$

$$\ln \frac{(V-40)}{60} = -\frac{1}{4}t$$

$$\frac{V-40}{60} = e^{-\frac{1}{4}t}$$

$$V = 60e^{-\frac{1}{4}t} + 40 \quad (7)$$

$$(c) \text{ as } t \rightarrow \infty \quad V \rightarrow 40. \quad (1)$$

$$8. (a) l \text{ and } m \text{ are perpendicular} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ 2 \end{pmatrix} = 0$$

$$b - 4 = 0$$

$$b = 4$$

$$l \text{ and } m \text{ intersect} \Rightarrow 2 = 1 + \mu a \quad \dots [A]$$

$$-1 + \lambda = -2 + \mu \times 4 \quad \dots [B]$$

$$3 - 2\lambda = -5 + \mu \times 2 \quad \dots [C]$$

$$\text{solving [B] and [C], } \lambda = 3, \mu = 1$$

$$\text{substitute in [A], } a = 1 \quad (6)$$

$$(b) \text{ point of intersection is } \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \quad (3)$$

$$(c) \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \sqrt{5}\sqrt{9}\cos\theta, \text{ where } \theta = \text{angle between lines}$$

$$0 + 2 - 4 = 3\sqrt{5}\cos\theta$$

$$\theta = 73^\circ \quad (3)$$
