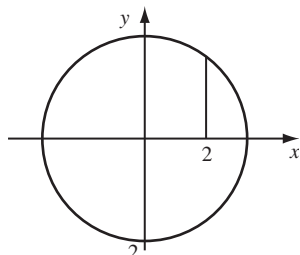


Worked Solutions

Edexcel C4 Paper D

1. (a)



$$(b) \text{ volume} = \pi \int_0^2 y^2 dx = \pi \int_0^2 (9 - x^2) dx$$

$$= \pi \left[9x - \frac{1}{3}x^3 \right]_0^2 = \pi \left(18 - \frac{8}{3} - 0 \right)$$

$$= \frac{46}{3}\pi \text{ cubic units.}$$

(2)

2. (a) $\cos^3 x = \cos x \cdot \cos^2 x$

$$= \cos x(1 - \sin^2 x) = \cos x - \cos x \sin^2 x$$

(1)

(b) $\frac{d}{dx}(\sin x)^3 = 3(\sin x)^2 \cdot \cos x$

(1)

(c) $\int (\cos x - \cos x \sin^2 x) dx = \sin x - \frac{1}{3} \sin^3 x + c$

(3)

3. Given $\frac{dS}{dt} = 640 \text{ cm}^2 \text{ s}^{-1}$. To find $\frac{dr}{dt}$.

$$S = 4\pi r^2$$

$$\frac{dS}{dr} = 8\pi r$$

$$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$$

when $r = 5$, $640 = 8\pi \times 5 \times \frac{dr}{dt}$

$$\frac{dr}{dt} = \frac{640}{40\pi}$$

$$= \frac{16}{\pi} \text{ cm s}^{-1}$$

(4)

4. (a) $2x + 2y \frac{dy}{dx} - 2 + 4 \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(2y + 4) = 2 - 2x$$

$$\frac{dy}{dx} = \frac{2 - 2x}{2y + 4}$$

$$= \frac{1 - x}{y + 2}$$

(2)

(b) at (4, 2) gradient of tangent = $\frac{1 - 4}{2 + 2} = -\frac{3}{4}$

equation of tangent is $y - 2 = -\frac{3}{4}(x - 4)$

$$4y + 3x = 20$$

(4)

$$5. (a) x = \frac{\ln 11}{\ln 5} \left(\text{or } \frac{\log 11}{\log 5} \right) = 1.49$$

$$(b) y = 3^x$$

$$\ln y = \ln 3^x$$

$$\ln y = x \ln 3$$

$$\frac{1}{y} \frac{dy}{dx} = \ln 3$$

$$\frac{dy}{dx} = y \ln 3 = 3^x \ln 3$$

(2)

$$(c) e^{-0.4x} = 5$$

$$\ln e^{-0.4x} = \ln 5$$

$$-0.4x = \ln 5$$

$$x = -\frac{\ln 5}{0.4} = -4.02$$

(2)

$$6. (a) y = x + 9x^{-1}$$

$$\frac{dy}{dx} = 1 - \frac{9}{x^2}$$

$$\text{at min. point } \frac{9}{x^2} = 1, \quad x = 3 \quad (x > 0)$$

$$\text{when } x = 3, \quad y = 3 + \frac{9}{3} = 6$$

$$\text{so } y \geq 6.$$

(4)

$$(b) \text{ area} = \int_3^9 \left(x + \frac{9}{x} \right) dx = \left[\frac{x^2}{2} + 9 \ln x \right]_3^9 = \frac{81}{2} + 9 \ln 9 - \left(\frac{9}{2} + 9 \ln 3 \right)$$

$$= 36 + 9 \ln 3.$$

(4)

$$7. (a) \frac{dy}{dt} = 4e^{4t} - 3, \quad \frac{dx}{dt} = 4e^{2t} - 1$$

$$\frac{dy}{dx} = \frac{4e^{4t} - 3}{4e^{2t} - 1}$$

(3)

$$(b) \text{ gradient} = 3, \quad \frac{4e^{4t} - 3}{4e^{2t} - 1} = 3$$

$$4e^{4t} - 3 = 12e^{2t} - 3$$

$$e^{4t} = 3e^{2t}$$

$$e^{2t} = 3$$

$$2t = \ln 3$$

$$t = \frac{1}{2} \ln 3$$

(5)

$$8. (a) P : 2\mathbf{i} + \mathbf{k}, \quad Q : \mathbf{j} + 2\mathbf{k}$$

(2)

$$(b) \vec{QP} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \text{ equation of line } QP \text{ is } r = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

(2)

$$(c) \text{ let } \angle OPQ = \theta$$

$$\vec{PO} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}, \vec{PQ} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, |\vec{PO}| = \sqrt{5}, |\vec{PQ}| = \sqrt{6}$$

$$\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = \sqrt{5}\sqrt{6} \cos \theta$$

$$3 = \sqrt{5}\sqrt{6} \cos \theta$$

$$\theta = 56.8^\circ$$

(3)

9. (a) (i) $\cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta = 1 + \sin 2\theta$

(ii)
$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 + \sin 2\theta) \, d\theta = \left[\theta - \frac{1}{2} \cos 2\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} - \frac{1}{2} \cos \pi - \left(\frac{\pi}{4} - \frac{1}{2} \cos \frac{\pi}{2} \right)$$

$$= \frac{\pi}{2} - \frac{1}{2}(-1) - \frac{\pi}{4} + 0 = \frac{\pi}{4} - \frac{1}{2}$$

(b)
$$\int_0^1 (2x+1)^4 \, dx = \left[\frac{1}{5} \cdot \frac{1}{2} (2x+1)^5 \right]_0^1 = \frac{1}{10} (3^5 - 1) = 24.2$$

10. (a)
$$\frac{3+5x-x^2}{(2-x)(1+x)^2} \equiv \frac{A(1+x)^2 + B(2-x)(1+x) + C(2-x)}{(2-x)(1+x)^2}$$

$3+5x-x^2 \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$

let $x = -1$, $3 - 5 - 1 = C(2 - (-1))$, $C = -1$

let $x = 2$, $3 + 10 - 4 = A(1 + 2)^2$, $A = 1$

constants, $3 = A + 2B + 2C$, $B = 2$

(b)
$$\int_0^1 \left(\frac{1}{2-x} + \frac{2}{1+x} - \frac{1}{(1+x)^2} \right) dx$$

$$= \left[-\ln(2-x) + 2\ln(1+x) + \frac{1}{(1+x)} \right]_0^1$$

$$= -\ln 1 + 2\ln 2 + \frac{1}{2} - (-\ln 2 + 2\ln 1 + 1) = 3\ln 2 - \frac{1}{2}$$

(2) (c)
$$\frac{1}{2-x} = \frac{1}{2\left(1-\frac{x}{2}\right)} = \frac{1}{2} \left(1-\frac{x}{2}\right)^{-1}$$

$$= \frac{1}{2} \left[1 + (-1)\left(-\frac{x}{2}\right) + \frac{(-1)(-2)}{2} \left(-\frac{x}{2}\right)^2 + \frac{(-1)(-2)(-3)}{3 \cdot 2} \left(-\frac{x}{2}\right)^3 + \dots \right]$$

$$= \frac{1}{2} \left[1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} \right]$$

$$= \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16}$$

(3)

(4)

$$f(x) = \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16} + 2(1+x)^{-1} - (1+x)^{-2}$$

$$= \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16}$$

$$+ 2 \left[1 + (-1)x + \frac{(-1)(-2)}{2} x^2 + \frac{(-1)(-2)(-3)}{3 \cdot 2} x^3 + \dots \right]$$

$$- \left[1 + (-2)x + \frac{(-2)(-3)}{2} x^2 + \frac{(-2)(-3)(-4)}{3 \cdot 2} x^3 + \dots \right]$$

$$= \left(\frac{1}{2} + 2 - 1 \right) + x \left(\frac{1}{4} - 2 + 2 \right) + x^2 \left(\frac{1}{8} + 2 - 3 \right) + x^3 \left(\frac{1}{16} - 2 + 4 \right)$$

$$= \frac{3}{2} + \frac{1}{4}x - \frac{7}{8}x^2 + \frac{33}{16}x^3.$$

(4)

(6)

(d) valid for $-1 < x < 1$.

(4)