mark scheme

Practice Paper A : Core Mathematics 4

<table>
<tr><th>Question Number</th><th colspan="2">General Scheme</th><th>Marks</th></tr>
<tr><td rowspan="4">1</td><td>$x^3-2x^2-5x+6=(x-1)(x+2)(x-3)$</td><td>M1 – a fully correct method to factorise the cubic (i.e. use of the factor theorem, long division etc).
A1 – correct factorisation</td><td>M1

A1</td></tr>
<tr><td colspan="2">Let
$$\frac{2\cancel{(x+2)}}{(x-1)\cancel{(x+2)}(x-3)}=\frac{2}{(x-1)(x-3)}=\frac{A}{x-1}+\frac{B}{x-3}$$
$$\therefore 2=A(x-3)+B(x-1)$$
B1 – simplifies the fraction by cancelling the common factor of $x+2$
M1 – correct method to separate the fraction into its parts</td><td>B1
M1</td></tr>
<tr><td>Let $x=3$: $2=2B \rightarrow B=1$

Let $x=1$: $2=-2A \rightarrow A=-1$</td><td>dM1 – correct attempt to find the values of A and B
A1 – one value correct</td><td>M1

A1</td></tr>
<tr><td>$$\therefore \frac{2x+4}{(x-1)(x+2)(x-3)}=\frac{1}{x-3}-\frac{1}{x-1}$$</td><td>A1 – cao, answers <u>must</u> be stated in this form at the end, otherwise award A0</td><td>A1</td></tr>
<tr><td></td><td colspan="2" align="right">Total</td><td>7</td></tr>
<tr><td>Note</td><td colspan="3">Some candidates may not cancel the $x+2$, but apply partial fractions to get
$$\frac{2x+4}{(x-1)(x+2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-3}+\frac{C}{x+2}$$
In this case, B1 becomes the third A1 which you should award for one correct value in addition to $C=0$.</td></tr>
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2 (a)	$f(x)=(4+x)^{-\frac{1}{2}}=\frac{1}{2}\left(1+\frac{1}{4}x\right)^{-\frac{1}{2}}$	**B1** – correct expression for $f(x)$	**B1**
	$f(x)\simeq\frac{1}{2}\left[\underbrace{1+\left(-\frac{1}{2}\right)\left(\frac{1}{4}x\right)}_{\text{B1}}+\underbrace{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{2!}\right)\left(\frac{1}{4}x\right)^{2}}_{\text{M1 A1}}+\ldots\right]$ **B1** – correct first two terms of the expansion **M1** – third term of the form $\frac{n(n-1)}{2!}x^2$, condone one slip **A1** – third term correctly expressed (need not be simplified for this mark)		**B1** **M1** **A1**
	$f(x)=\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2+\ldots$	**A1** – cao, expansion fully simplified	**A1** **(5)**
(b)	$4+x=7 \ \rightarrow\ x=3$ $\frac{\sqrt{7}}{7}=\frac{1}{2}-\frac{1}{16}(3)+\frac{3}{256}(3)^2+\ldots$ $\sqrt{7}=2.9257\ldots$	**B1** – correct value of x **M1** – substitutes 3 into their expansion **A1ft** – correct estimate of $\sqrt{7}$ using *their* expansion	**B1** **M1** **A1** **(3)**
(c)	(It is correct to) one significant figure	**B1** – cao, no ft	**B1** **(1)**
	Total		**9**

3 (a)	$\int_0^{\frac{\pi}{4}} \tan x \, dx = \left[-\ln\lvert\cos x\rvert\right]_0^{\frac{\pi}{4}}$	**B1** – correct integral	**B1**
	$= -\ln\left\lvert\cos\frac{\pi}{4}\right\rvert - -\ln\lvert\cos 0\rvert$	**M1** – substitutes limits the right way round	**M1**
	$= \ln\left(\frac{\sqrt{2}}{2}\right)^{-1} = \ln\left(\frac{2}{\sqrt{2}}\right) = \ln\left(\sqrt{2}\right)$	**M1** – uses the rule that $-\ln x = \ln\left(\frac{1}{x}\right)$ **A1** – cao **AG**	**M1** **A1** **(4)**
(b)	$x = \sin u \;\rightarrow\; dx = \cos u \; du$ $\therefore \int_0^{\frac{\sqrt{2}}{2}} \frac{5x}{2-2x^2} = \frac{5}{2}\int_0^{\frac{\pi}{4}} \frac{\sin u}{\cos^2 u}(\cos u)\,du = \frac{5}{2}\int_0^{\frac{\pi}{4}} \tan u \; du$	**M1** – substitutes $\sin u$ in replace of x **B1** – correct limits **A1** – integral becomes $k\int_0^{\frac{\pi}{4}} \tan u \; du$	**M1** **B1** **A1**
	$= \frac{5}{2}\ln\left(\sqrt{2}\right)$	**A1** – cao	**A1** **(4)**
		Total	**8**

4 (a)	Let $y = a^x$ $\ln y = x \ln a$	**M1 –** use of logarithims	**M1**
	Differentiate wrt x : $\frac{1}{y}\frac{dy}{dx} = \ln a$	**A1 –** correct implicit differentiation	**A1**
	$\frac{dy}{dx} = y \ln a = a^x \ln a$	**A1** – cso **AG**	**A1** **(3)**
(b)	$2^x \ln 2 - 4y\frac{dy}{dx} = x\frac{dy}{dx} + y$	**B1** – correct differentiation on LHS **M1** – application of the product rule on RHS	**B1** **M1**
	$(x+4y)\frac{dy}{dx} = 2^x \ln 2 - y$	**M1** – rearranges for $\frac{dy}{dx}$	**M1**
	$\frac{dy}{dx} = \frac{2^x \ln 2 - y}{x + 4y}$	**A1** – correct differentiation **OE**	**A1** **(4)**
	Total		**7**

5 (a)	$\sin 3t = 0 \;\rightarrow\; t = 0, \dfrac{\pi}{3}, \ldots$	**B1** – correct parameters	**B1**
	$y = \dfrac{2}{2-\cos 3t}$ $\therefore y = \dfrac{2}{2-\cos 0}, \dfrac{2}{2-\cos \pi}$	**M1** – rearranges for y **and** substitutes *their* parameters to find A and B	**M1**
	$A(0,2)$, $B\left(0, \dfrac{2}{3}\right)$	**A1** – cao	**A1** **(3)**
(b)	$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$	**M1 –** this principle used or attempted at any point in the question	**M1**
	$\dfrac{dy}{dx} = \dfrac{-2(3\sin 3t)}{(2-\cos 3t)^2} \times \dfrac{1}{3\cos 3t}$	**M1** – use of the quotient rule to find $\dfrac{dy}{dt}$ **B1** – correct $\dfrac{dt}{dx}$	**M1** **B1**
	$\dfrac{dy}{dx} = \dfrac{-2(3\sin 3t)}{(2-\cos 3t)^2} \times \dfrac{\tan 3t}{3\cos 3t}$	**M1** – attempts to simplify answer into required form using $\dfrac{\sin x}{\cos x} = \tan x$	**M1**
	$\dfrac{dy}{dx} = -\dfrac{2\tan 3t}{(2-\cos 3t)^2}$	**A1** – cao **AG**	**A1** **(5)**
(c)	$x = \dfrac{\sqrt{3}}{2} \rightarrow t = \dfrac{\pi}{9}$	**B1** – correct parameter	**B1**
	$\left.\dfrac{dy}{dx}\right\|_{t=\frac{\pi}{9}} = -\dfrac{2\tan\left(\frac{\pi}{3}\right)}{\left(2-\cos\left(\frac{\pi}{3}\right)\right)^2}$ $= -\dfrac{2\sqrt{3}}{\frac{9}{4}} = -\dfrac{8\sqrt{3}}{9}$	**M1** – substitutes *their* parameter into $\dfrac{dy}{dx}$ **A1** – $-\dfrac{8\sqrt{3}}{2}$	**M1** **A1**
	$y - \dfrac{4}{3} = -\dfrac{8\sqrt{3}}{9}\left(x - \dfrac{\sqrt{3}}{2}\right)$	**A1ft** – equation of tangent **OE**	**M1** **(4)**

(d)	$y-\frac{4}{3}=\frac{8\sqrt{3}}{9}\left(x+\frac{\sqrt{3}}{2}\right)$	**B1** – cao	**B1** **(1)**
(e)	$P\left(\sqrt{3},0\right)$, $Q\left(-\sqrt{3},0\right)$	**B2** – coordinates of P and Q (**B1** for each one)	**B2**
	$AP=\sqrt{\left(\sqrt{3}\right)^2+2^2}=\sqrt{7}$ $AQ=\sqrt{\left(-\sqrt{3}\right)^2+2^2}=\sqrt{7}$ $\therefore AP=AQ$	**M1** – use of Pythagoras to work out AP or AQ **A1** – correctly shows that $AP=AQ=\sqrt{7}$	**M1** **A1** **(4)**
	Total		**17**

6	$6+4\mu=5 \ \rightarrow \ \mu=-\frac{1}{4}$	**B1** – correct μ	**B1**
	$1+\left(-\frac{1}{4}\right)a=\frac{3}{4}$ $a=1$	**M1** – attempt to find a **A1** – correct a	**M1** **A1**
	$a+\lambda=\frac{3}{4} \ \rightarrow \ \lambda=-\frac{1}{4}$	**M1** – attempts to find λ or 9λ	**M1**
	$\therefore 3-\frac{b}{4}=-\frac{11}{4}$ $\frac{b}{4}=\frac{23}{4}$ $b=23$	**M1** – attempts to find b **A1** – correct b	**M1** **A1**
		Total	**6**

7 (a)	$y=\frac{9\sqrt{2}}{4\pi}\left(\frac{\pi}{6}\right)=\frac{3\sqrt{2}}{8}$ M1 $y=\cos^2\left(\frac{\pi}{6}\right)\sqrt{\sin\left(\frac{\pi}{6}\right)}=\frac{3}{4}\times\frac{1}{\sqrt{2}}=\frac{3\sqrt{2}}{8}$	**M1** – attempts to show that the coordinates satisfy one equation **A1** – shows *thoroughly* that the coordinates satisfy **both** equations	**M1** **A1** **(2)**
(b)	$\pi\int_0^{\frac{\pi}{6}}\left(\cos^2 x\sqrt{\sin x}\right)^2 dx=\pi\int_0^{\frac{\pi}{6}}\cos^4 x\sin x\, dx$	**B1** – use of $\pi\int_a^b y^2 dx$, with a and b correct	**B1**
	$=\pi\left[-\frac{\cos^5 x}{5}\right]_0^{\frac{\pi}{6}}$ $=\pi\left[-\frac{9\sqrt{3}}{5}--\frac{1}{5}\right]$ $=\frac{\pi\left(1-9\sqrt{3}\right)}{5}$	**M1** – $\frac{k\cos^5 x}{5}$ seen **A1** – correct integration **dM1** – substitutes *their* limits the right way around **A1ft** – correct value for the volume of revolution of the curve	**M1** **A1** **M1** **A1**
	Volume of revolution of line $=\frac{1}{3}\pi\left(\frac{3\sqrt{2}}{4}\right)^2\left(\frac{\pi}{6}\right)=\frac{\pi^2}{16}$ Volume of revolved shaded region $=\frac{\pi\left(1-9\sqrt{3}\right)}{5}-\frac{\pi^2}{16}$	**B1** – correct value for volume of the revolved line **ddM1** – *their* volume of revolved curve – *their* volume of revolved line **A1** – cao oe	**B1** **M1** **A1** **(8)**
	Total		**10**

8 (a)	$\frac{dx}{dt} = kx$	**B1** – forms a differential equation of the correct form	**B1**
	$\frac{1}{x}\frac{dx}{dt} = k$ $\int \frac{1}{x}\frac{dx}{dt}dt = \int k \ dt$ $\int \frac{1}{x} \ dx = \int k\,dt$	**M1** – method to separate the variables **A1** – variables correctly separated	**M1** **A1**
	$\ln x = kt + c$	**A1** – correct integration <u>including constant</u>	**A1**
	$x = e^{kt+c} = Ae^{kt}$ $450 = Ae^{k(0)} \ \rightarrow \ A = 450$ $2103 = 450e^{k(4)}$ $k = \frac{1}{4}\ln\left(\frac{2103}{450}\right)$	**dM1** – attempt to find constants **A1ft** – one constant correctly found, ft *their* integration and separation of variables	**M1** **A1**
	$x = 450e^{\frac{1}{4}\ln\left(\frac{2103}{450}\right)\times 7} = 6684.35$	**M1** – substitutes 7 into *their* solution to the DE	**M1**
	There will be about 6680 strands of bacteria in the culture after one week.	**A1** – correct answer given in context	**A1** **(8)**
(b)	**B1** – number of bacteria, x, on vertical axis, time, t, on horizontal axis **B1** – correct shape (showing exponential growth) **B1** – the curve starts from 450 on the vertical axis. Award **B1 B1 B0** for graphs that show negative time		**B1** **B1** **B1** **(3)**
	Total		**11**

Notes on alternative methods:

This mark scheme may feature some alternative solutions, but, of course, at this level, there is likely to be questions that have many others. Where alternative methods are used, you should award full marks **if the method is correct** (do **not** award full marks for methods that coincidentally lead to the right answer). If the method is *not* correct, then you should aim to mark it by being as faithful to the original scheme as you can and ensure that you award the same amount of marks for the same amount of *progress* in a question as you would award using the general scheme.