FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper I

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper I – Marking Guide

1.		$3x^2 + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	M1 A2	
		$(2, -4) \Rightarrow 12 - 8 + 4\frac{dy}{dx} + 8\frac{dy}{dx} = 0, \quad \frac{dy}{dx} = -\frac{1}{3}$	M1 A1	
		grad of normal = 3	M1	
		$\therefore\ y + 4 = 3(x - 2)$	M1	
		$y = 3x - 10$	A1	**(8)**

2.	*(a)*	$= 4^{\frac{1}{2}}(1 - \frac{1}{4}x)^{\frac{1}{2}} = 2(1 - \frac{1}{4}x)^{\frac{1}{2}}$	B1	
		$= 2[1 + (\frac{1}{2})(-\frac{1}{4}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-\frac{1}{4}x)^2 + \ldots] = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \ldots$	M1 A2	
	(b)	$\lvert x \rvert < 4$	B1	
	(c)	$x = 0.01 \Rightarrow (4 - x)^{\frac{1}{2}} = \sqrt{3.99} = \sqrt{\frac{399}{100}} = \frac{1}{10}\sqrt{399}$	M1	
		$x = 0.01 \Rightarrow (4 - x)^{\frac{1}{2}} \approx 2 - \frac{1}{400} - \frac{1}{640000} = 1.997498438$	M1	
		$\therefore\ \sqrt{399} \approx 10 \times 1.997498438 = 19.9749844$ (9sf)	M1 A1	**(9)**

3.	*(a)*	0.9959, 0.6931, 0.2569 (4dp)	B2	
	(b)	*(i)* $= \frac{1}{2} \times \pi \times (1.0986 + 0) = 1.726$ (3dp)	B1 M1 A1	
		(ii) $= \frac{1}{2} \times \frac{\pi}{2} \times [1.0986 + 0 + 2(0.6931)] = 1.952$ (3dp)	M1 A1	
		(iii) $= \frac{1}{2} \times \frac{\pi}{4} \times [1.0986 + 0 + 2(0.9959 + 0.6931 + 0.2569)]$		
		$= 1.960$ (3dp)	A1	
	(c)	1.96; large change from 1 to 2 strips but from 2 to 4 strips the change is less than 0.01 so the error in 4 strip value is likely to be less than 0.005	B2	**(10)**

4.	*(a)*	$x = -1 \Rightarrow \theta = -\frac{\pi}{4},\ x = 1 \Rightarrow \theta = \frac{\pi}{4}$	B1	
		$\frac{dx}{d\theta} = \sec^2\theta$	M1	
		volume $= \pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\cos^2\theta)^2 \times \sec^2\theta\ d\theta = \pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2\theta\ d\theta$	A1	
		$= \pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\frac{1}{2} + \frac{1}{2}\cos 2\theta)\ d\theta$	M1	
		$= \pi[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$	M1 A1	
		$= \pi[(\frac{\pi}{8} + \frac{1}{4}) - (-\frac{\pi}{8} - \frac{1}{4})]$	M1	
		$= \pi(\frac{\pi}{4} + \frac{1}{2}) = \frac{1}{4}\pi(\pi + 2)$	A1	
	(b)	$y = \cos^2\theta = \frac{1}{\sec^2\theta} = \frac{1}{1+\tan^2\theta} \quad \therefore\ y = \frac{1}{1+x^2}$	M2 A1	**(11)**

5.	*(a)*	$\overrightarrow{AB} = (5\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (3\mathbf{i} - 3\mathbf{j} - 6\mathbf{k})$	B1	
		$\overrightarrow{AC} = (7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (5\mathbf{i} - 5\mathbf{j} - 10\mathbf{k}) = \frac{5}{3}\overrightarrow{AB}$	M1	
		$\therefore$ $\overrightarrow{AC}$ is parallel to $\overrightarrow{AB}$, also common point $\therefore$ single straight line	A1	
	(b)	3 : 2	B1	
	(c)	$\overrightarrow{AD} = (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	B1	
		$\overrightarrow{BD} = (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (5\mathbf{i} - 4\mathbf{j}) = (-2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$	B1	
		$\overrightarrow{AD} \cdot \overrightarrow{BD} = -2 + 10 - 8 = 0 \;\; \therefore$ perpendicular	M1 A1	
	(d)	$= \frac{1}{2} \times \sqrt{1+4+4} \times \sqrt{4+25+16} = \frac{1}{2} \times 3 \times 3\sqrt{5} = \frac{9}{2}\sqrt{5}$	M2 A1	**(11)**

6.	*(a)*	$x = 2\sin u \Rightarrow \frac{dx}{du} = 2\cos u$	M1	
		$x = 0 \Rightarrow u = 0, \; x = \sqrt{3} \Rightarrow u = \frac{\pi}{3}$	B1	
		$I = \int_0^{\frac{\pi}{3}} \frac{1}{2\cos u} \times 2\cos u \; du = \int_0^{\frac{\pi}{3}} 1 \; du$	A1	
		$= [u]_0^{\frac{\pi}{3}} = \frac{\pi}{3} - 0 = \frac{\pi}{3}$	M1 A1	
	(b)	$u = x, \; u' = 1, \; v' = \cos x, \; v = \sin x$	M1	
		$I = [x\sin x]_0^{\frac{\pi}{2}} - \int \sin x \; dx$	A2	
		$= [x\sin x + \cos x]_0^{\frac{\pi}{2}}$	M1	
		$= (\frac{\pi}{2} + 0) - (0 + 1) = \frac{\pi}{2} - 1$	M1 A1	**(11)**

7.	*(a)*	when $x = \frac{1}{4}, \; \frac{dx}{dt} = \frac{3}{4} \div 6 = \frac{1}{8}$	M1 A1	
		$\frac{dx}{dt} = kx(1 - x) \;\; \therefore \; \frac{1}{8} = k \times \frac{1}{4} \times \frac{3}{4}, \; k = \frac{2}{3} \;\; \therefore \; \frac{dx}{dt} = \frac{2}{3}x(1 - x)$	M1 A1	
	(b)	$\int \frac{1}{x(1-x)} \; dx = \int \frac{2}{3} \; dt$	M1	
		$\frac{1}{x(1-x)} \equiv \frac{A}{x} + \frac{B}{1-x}, \qquad 1 \equiv A(1 - x) + Bx$	M1	
		$x = 0 \quad \Rightarrow \quad A = 1$	A1	
		$x = 1 \quad \Rightarrow \quad B = 1$	A1	
		$\therefore \; \int (\frac{1}{x} + \frac{1}{1-x}) \; dx = \int \frac{2}{3} \; dt$		
		$\ln\lvert x \rvert - \ln\lvert 1 - x \rvert = \frac{2}{3}t + c$	M1 A1	
		$t = 0, \; x = \frac{1}{4} \Rightarrow \ln\frac{1}{4} - \ln\frac{3}{4} = c, \quad c = \ln\frac{1}{3}$	M1 A1	
		$t = 3 \Rightarrow \ln\lvert x \rvert - \ln\lvert 1 - x \rvert = 2 + \ln\frac{1}{3}$		
		$\ln\left\lvert \frac{3x}{1-x} \right\rvert = 2, \qquad \frac{3x}{1-x} = e^2$	M1	
		$3x = e^2(1 - x), \quad x(e^2 + 3) = e^2$	M1	
		$x = \frac{e^2}{e^2+3} \;\; \therefore$ % destroyed $= \frac{e^2}{e^2+3} \times 100\% = 71.1\%$ (3sf)	A1	**(15)**

Total **(75)**

Performance Record – C4 Paper I

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	differentiation	binomial series	trapezium rule	parametric equations	vectors	integration	differential equation, partial fractions	
Marks	8	9	10	11	11	11	15	75
Student								