

Question Number	Scheme	Marks
<p><b>1.</b> (a)</p> <p>(b)</p>	<p><math>p = 1.357; q = 1.382</math></p> <p><math>I \approx \frac{0.5}{2} [1 + 2(1.216 + 1.357 + 1.413) + 1.382]</math></p> <p><math>= 2.589</math></p>	<p>B1 B1 (2)</p> <p>B1 M1 A1 ft</p> <p>A1 (4)</p> <p><b>(6 marks)</b></p>
<p><b>2.</b> (a)</p> <p>(b)</p>	<p><math>\int x \cos 2x dx = \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} dx</math> (integration in correct direction)</p> <p><math>= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} (+k)</math> (second integration)</p> <p><math>x \frac{2 \sin x \cos x}{2} + \frac{1 - 2 \sin^2 x}{4} (+k)</math> (use of appropriate double angle formulae)</p> <p><math>= \frac{1}{2} \sin x (2x \cos x - \sin x) + \frac{1}{4} + k</math> for <math>\frac{1}{4} + k</math></p> <p><math>= \frac{1}{2} \sin x (2x \cos x - \sin x) + C \star</math></p>	<p>M1 A1</p> <p>M1 A1 (4)</p> <p>M1</p> <p>A1</p> <p>A1 cao (3)</p> <p><b>(7 marks)</b></p>
<p><b>3.</b> (a)</p> <p>(b)</p>	<p><math>(1 + 3x)^{-2} = 1 + (-2)(3x) + \frac{(-2)(-3)}{2!}(3x)^2 + \frac{(-2)(-3)(-4)}{3!}(3x)^3 + \dots</math></p> <p><math>= 1, -6x, +27x^2 \dots (-108x^3)</math></p> <p>Using (a) to expand <math>(x + 4)(1 + 3x)^{-2}</math> or complete method to find coefficients</p> <p>[e.g. Maclaurin or <math>\frac{1}{3}(1 + 3x)^{-1} + \frac{11}{3}(1 + 3x)^{-2}</math>].</p> <p><math>= 4 - 23x, +102x^2, -405x^3 = 4, -23x, +102x^2 \dots (-405x^3)</math></p>	<p>M1</p> <p>B1 A1 A1 (4)</p> <p>M1</p> <p>A1, A1ft,</p> <p>A1ft (4)</p> <p><b>(8 marks)</b></p>

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<p>4. (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p><math>\vec{AB} = 3\mathbf{b} + 6\mathbf{j} + 6\mathbf{k}</math></p> <p><math>\cos A = \frac{-12 - 48 + 6}{\sqrt{81}\sqrt{81}} = -\frac{2}{3}</math></p> <p><math>\lambda = 4</math> at point <math>A</math> and <math>\lambda = 7</math> at point <math>B</math>.</p> <p><math>\mathbf{r} = -9\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})</math> represents a line</p> <p><math>(\lambda\mathbf{i} + 2\lambda\mathbf{j} + (2\lambda - 9)\mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0</math></p> <p><math>\lambda + 4\lambda + \lambda - 18 = 0</math>. Therefore <math>\lambda = 2</math></p> <p>The point is <math>(2, 4, -5)</math></p>	<p>B1 (1)</p> <p>M1 A1 A1 (3)</p> <p>B1 B1</p> <p>B1 (3)</p> <p>M1</p> <p>M1 A1 (3)</p> <p>M1 A1 (2)</p> <p><b>(12 marks)</b></p>
<p>5. (a)</p> <p>(b)</p>	<p><math>\frac{dy}{dx} = \sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x</math></p> <p>At <math>A</math> <math>\sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x = 0</math></p> <p><math>\therefore \sin x + \frac{x}{2} \cos x = 0</math> (essential to see intermediate line before given answer)</p> <p><math>\therefore 2 \tan x + x = 0</math> *</p> <p><math>V = \pi \int y^2 dx = \pi \int x^2 \sin x dx</math></p> <p><math>= \pi \left[ -x^2 \cos x + \int 2x \cos x dx \right]_0^\pi</math></p> <p><math>= \pi \left[ -x^2 \cos x + 2x \sin x - \int 2 \sin x dx \right]_0^\pi</math></p> <p><math>= \pi \left[ -x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^\pi</math></p> <p><math>= \pi \left[ \pi^2 - 2 - 2 \right]</math></p> <p><math>= \pi \left[ \pi^2 - 4 \right]</math></p>	<p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (7)</p> <p><b>(11 marks)</b></p>

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<b>6.</b> (a)  (b)  (c)  (d)	$\frac{dN}{dt} = -kN$ $\int \frac{dN}{N} = \int -k dt$ $\ln N = -kt + c$ $N = e^{-kt+c} = Ae^{-kt}$ $3 \times 10^{17} = 7 \times 10^{18} e^{-8k}$ $e^{-k} = \sqrt[8]{\frac{3}{70}} = 0.6745 \quad \text{or} \quad k = \frac{1}{8} \ln \frac{70}{3}$ $k = 0.3937$ $N = 7 \times 10^{18} e^{-0.3937 \times 16} \quad \text{or} \quad \frac{3}{70} \times 3 \times 10^{17}$ $= 1.286 \times 10^{16}$	M1 A1 (2)  B1 ft  M1 A1 ft M1 A1 (5)  M1  M1  A1 (3)  M1  A1 (2)  <b>(12 marks)</b>
<b>7.</b> (a)  (b)	$A = 2, \quad B = -16$ $A(1 - 2x)^{-1} + B(2 + x)^{-1} \quad \text{and attempt at expansion}$ $A(1 + 2x + 4x^2 + 8x^3 + \dots) + \frac{B}{2} \left(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \dots\right)$ $= 10 + 10x^2 + 15x^3 + \dots$	M1 A1 A1 (3)  M1  A1 M1 A1  A1 (5)  <b>(8 marks)</b>

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<p><b>8.</b></p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{4 \cos \theta}{-5 \sin \theta}$ <p>Equation of tangent is <math>y - 4 \sin \alpha = \frac{4 \cos \alpha}{-5 \sin \alpha} (x - 5 \cos \alpha)</math></p> <p><math>\therefore 5y \sin \alpha + 4x \cos \alpha = 20(\cos^2 \alpha + \sin^2 \alpha) = 20 \quad (*)</math></p> $\int y \frac{dx}{d\theta} d\theta = - \int 4 \sin \theta \cdot 5 \sin \theta \, d\theta$ $= 10 \int (\cos 2\theta - 1) \, d\theta$ $= [5 \sin 2\theta - 10\theta]$ <p>Area = <math>20\pi</math></p> <p>When <math>x = 0, y = \frac{4}{\sin \alpha}</math>, or when <math>y = 0, x = \frac{5}{\cos \alpha}</math></p> <p>Area of parallelogram = <math>4 \times \frac{10}{\sin \alpha \cos \alpha} = \frac{80}{\sin 2\alpha}</math></p> <p><math>\therefore A = \frac{80}{\sin 2\alpha} - 20\pi</math></p> $\frac{80}{\sin 2\alpha} - 20\pi = 20\pi$ $\sin 2\alpha = \frac{2}{\pi}$ <p><math>\alpha = 0.345</math></p>	<p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>M1</p> <p>A1 cso (4)</p> <p>B1</p> <p>M1 A1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>A1 (3)</p> <p><b>(15 marks)</b></p>